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AMERICAN SOCIETY
OF CIVIL ENGINEERS

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Journal of the

STRUCTURAL DIVISION

Proceedings of the American Society of Civil Engineers

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Journal of the
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LESSONS OF COLLAPSE OF VANCOUVER 2nd NARROWS BRIDGE

A. Hrennikoff,[1] F. ASCE

INTRODUCTION

In this paper the writer presents some conclusions from his study of the collapse of the Second Narrows Bridge in Vancouver, B. C. in the summer of 1958. He sees in this tragic occurrence not only a mere consequence of an error in design but also a manifestation of the effects of some factors not yet fully appreciated. Realization of their true significance may be one positive aspect of the disaster.

General

The collapse of the Second Narrows Bridge was described in the February 1959 issue of "Civil Engineering" by Dr. F. M. Masters, M. ASCE, and J. R. Giese, M. ASCE, of Modjeski and Masters, Harrisburg, Pa., who, together with J. R. H. Otter of Rendel, Palmer and Tritton, and Ralph Freeman, M. ASCE, of Freeman, Fox and Partners, both of London, England, and A. B. Sanderson of Victoria, B. C., were the principal engineering advisers to the Royal Commission under the Hon. Chief Justice Sherwood Lett, appointed by the Government of British Columbia to investigate the disaster. The writer was engaged by the Royal Commission for the study of one particular part of the bridge, the grillage of the falsework bent supporting the bridge during the erection which proved to be the cause of the collapse. The lessons which the writer discerns in the catastrophe are:

1. The inapplicability of the usual column formulae for the design of the webs of the grillage beams in buckling.
2. The weakening effect of the plywood pads interposed in the grillage.
3. The general inadequacy of the column formula given by the Canadian Specifications CSA, 1952.

Note: Discussion open until May 1, 1960. To extend the closing date one month, a written request must be filed with the Executive Secretary, ASCE. Paper 2305 is part of the copyrighted Journal of the Structural Division, Proceedings of the American Society of Civil Engineers, Vol. 85, No. ST 10, December, 1959.

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Prior to the discussion of these matters it is necessary to describe the construction of the grillage and its status with regard to the buckling failure.

Construction of the Grillage

The falsework bent supported by the grillage, which caused the failure, is shown in the photograph (Fig. 1) taken shortly before the collapse. The grillage itself is presented in Fig. 2. The two columns of the bent, with the base plates 5 3/4" thick underneath them, were supported by two tiers of beams which rested on piles. The upper tier of four beams or stringers 36" W at 160# extended transversely to the bridge from one column to the other and was supported under each column by a group of four built-up, welded, grillage beams. These beams rested on open-end pipe piles filled with concrete. Plywood pads, 3/4" thick, were interposed between the column base and the stringers, and likewise between the latter and the grillage beams. The weakness of the grillage was due to the lack of diaphragms or adequate stiffeners in the upper tier stringers, which in the opinion of the Royal Commission led to the buckling of the webs of the stringers precipitating the collapse of the falsework bent and with it the bridge itself. (It may be mentioned that the Royal Commission discovered also some computation errors in the design of the stringers, consistent with the conclusion attributing failure to the buckling of the stringer webs.) The bottom tier grillage beams were provided with proper diaphragms and were fully adequate. The piles underneath them were also amply strong and rigid.

Buckling of Webs of Grillage Stringers

As may be seen in Fig. 2, the four stringers were held together by four packs of wooden blocking two of which were located just outside the falsework columns and the two others at the third points of the distance between the column centres. The ends of the two outer stringers were also connected by vertical angle brackets to the diaphragms on the grillage beams.

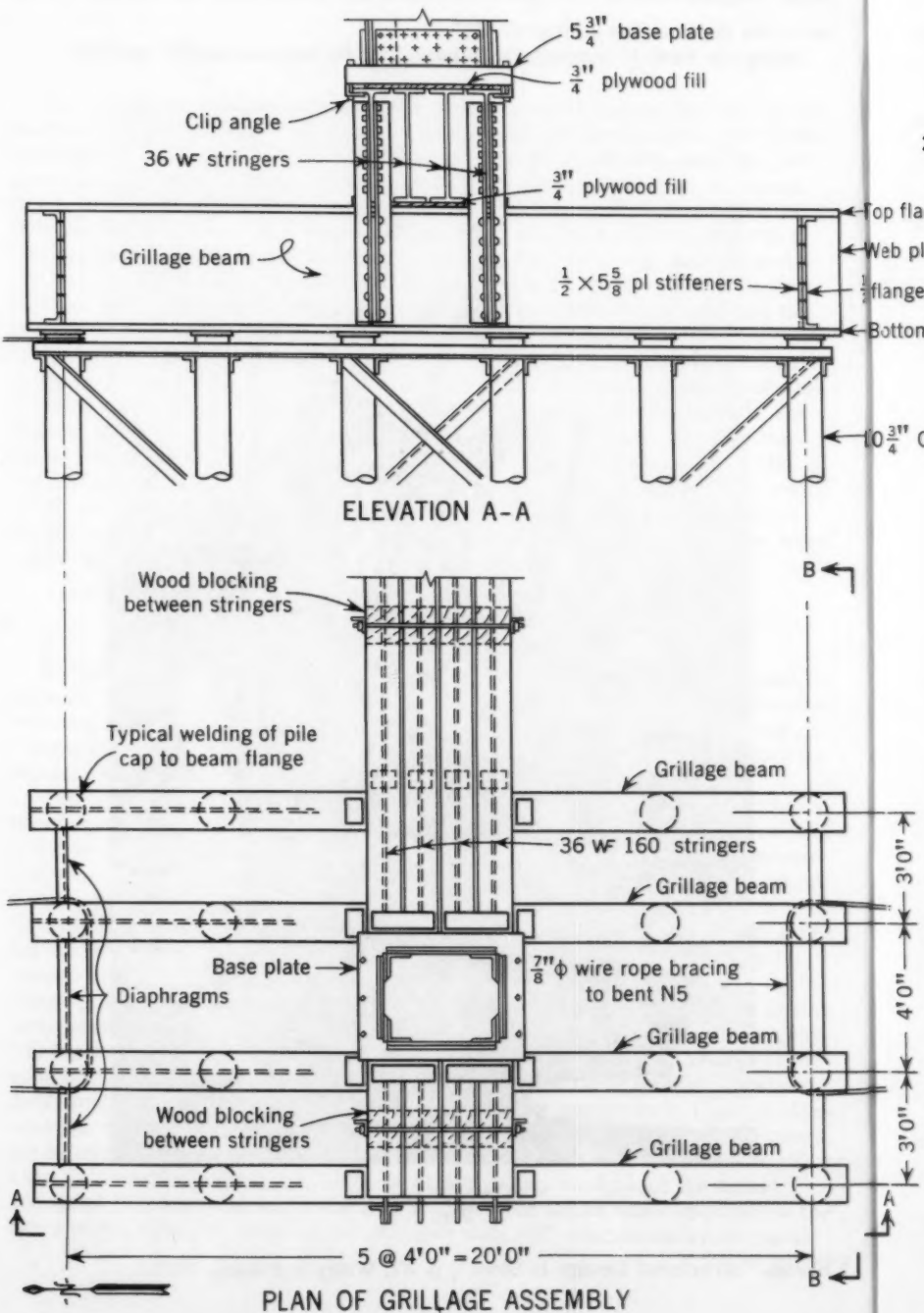
The buckling of the webs of the stringers occurred underneath the column bases in the manner of Fig. 3. The bottom flanges of the stringers were held in place by the friction on the grillage beams, while their top flanges moved sideways parallel to themselves together with the column base, as the column rocked about the hinge at its top end, underneath the truss. With the top flanges fixed in direction but free to move sideways, the structural action of the stringer webs in the course of buckling was equivalent to a pin ended column (Fig. 4) of the length L = 32 1/4" equal to the distance between the flange fillets. In this action the stringers were grossly underdesigned having their slenderness ratio 171 against the allowable 120 for main members and 140 for the secondary ones according to the column formula of AASHO 1955 specification (Curve 1, Fig. 13). Even using a much more lenient Canadian Specification, CSA 1952, (curve 2, Fig. 13) with the permissible limit of slenderness ratio 140 for the main members and 200 for the secondary members, the allowable stress using the regular column formula with a 33% extra allowance for erection conditions, came to 11.2 kips/sq. in. against the actual stress of 17.2 kips/sq. in. (point X, Fig. 13). The latter was computed on the basis of the effective width of the stringer web, 60", calculated by the usual

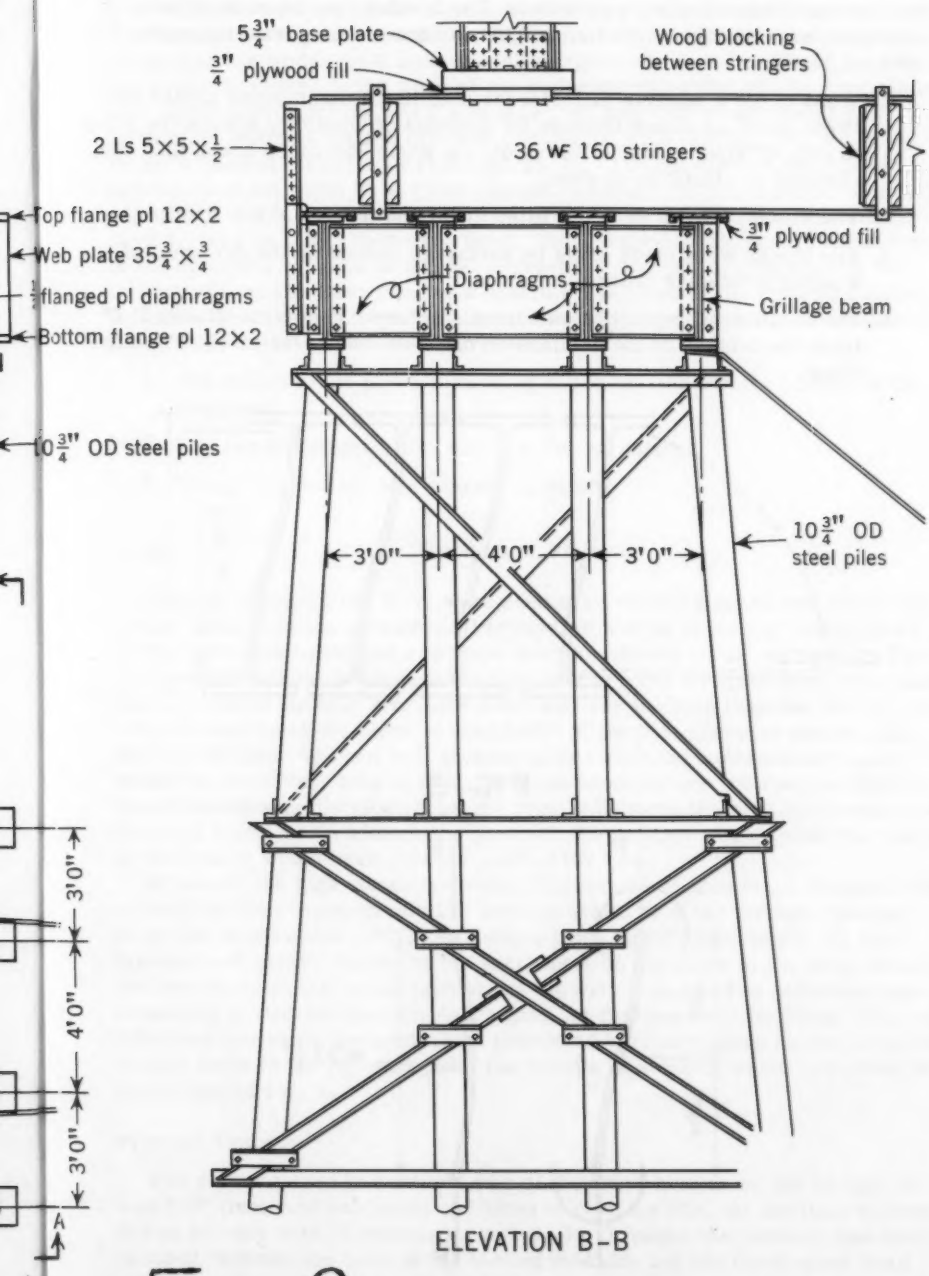
empirical formula* $w = b + \frac{d}{2}$, where b was the width of the column base, 42", and d the depth of the stringers, 36".

Being too weak to support the column load by themselves the grillage

Fig. 1.

* Shedd. "Structural Design in Steel", p. 87, Wiley and Sons, 1934.


5 3/4" base plate
3/4" plywood fill
Clip angle
36 WF stringers
3/4" plywood fill
Grillage beam
1/2 × 5 5/8 pl stiffeners
Top flange
Web plate
Bottom fla
10 3/4" OD s
ELEVATION A-A
B
Wood blocking between stringers
Typical welding of pile cap to beam flange
Grillage beam
36 WF 160 stringers
Grillage beam
Base plate
Diaphragms
7/8"φ wire rope bracing to bent N5
Grillage beam
Wood blocking between stringers
Grillage beam
A
A
3'0"
4'0"
3'0"
5 @ 4'0" = 20'0"
B
PLAN OF GRILLAGE ASSEMBLY



ELEVATION B-B

Fig. 2

beams needed rigid diaphragms or stiffeners for their stabilization. However no such members were provided. The brackets on the ends of the stringers were totally ineffective and the wooden blocking was inadequate for several reasons:

1. Wood is not a suitable material for stabilizing diaphragms placed between steel members because its modulus of elasticity across the grain is only 40 kips/sq. in. at the most, i.e. some 750 times lower than the modulus of elasticity of steel.
2. The wooden blocks were not fitted under the flanges of the stringers.
3. The blocks were made loose by shrinkage caused by the dry spell of weather preceding failure.
4. The location of the blocks was unsatisfactory. They were placed 3' 6" from the centres of the columns on one side and as far as 16' 0" on the other.

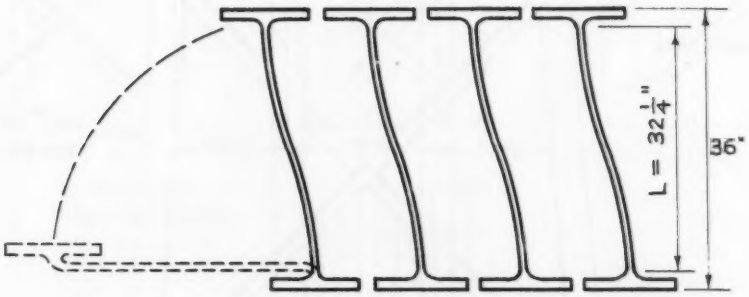


FIG. 3

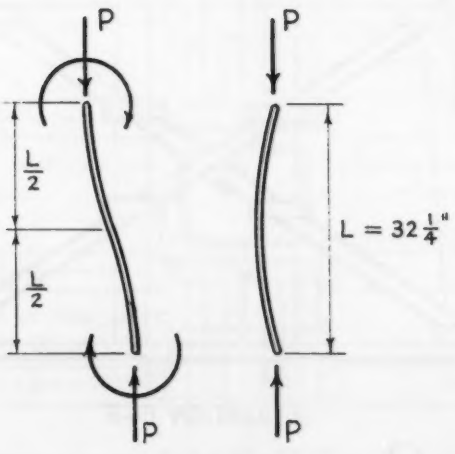


FIG. 4

These considerations suggest that the effectiveness of the wooden blocks in stabilizing the webs of the stringers was uncertain and probably small.

Although the use of wooden blocking in lieu of rigid diaphragms was a grave mistake of design it does not fully explain the failure. The allowable compression stress in the webs of the stringers using the column formula of CSA, 1952 specifications was 8.4 kips/sq. in. and with an additional 33% allowance, common for the erection conditions, 11.2 kips/sq. in., while the actual stress at failure was 17.2 kips/sq. in. The spread between these figures does not seem to be great enough to exceed the usual factor of safety especially if some positive strengthening effect is attributed to the wooden blocks, weak as they might have been. This reasoning points on the one hand to the presence of some additional aggravating factors contributory to the failure, and on the other hand to a possible weakness of the Canadian column formula. Both these conjectures are believed to be correct. The additional factors which influenced the failure were:

1. The action of the plywood packing at the top and bottom flanges of the stringers.
2. The imperfections of the shape of the stringers.
3. The difference of the stringers in depth.

The Role of Plywood

Pads of plywood 3/4" thick were used over the top flanges and under the bottom flanges of the stringers. Plywood, by virtue of its high compressibility, had some beneficial and some harmful effects on the stringers. Plywood was beneficial because it levelled off the local irregularities of contacts. Thus, if one of the four stringers were smaller in depth than the others, the compressibility of the plywood would still allow it to carry an appreciable part of the load. Without soft packing of this kind the undersize stringer would be completely free of load. For the same reason the load carried by the stringers was distributed nearly equally between the grillage beams underneath them, while without the plywood, the grillage beams near the centre of the footing would have carried most of the load.

However, the high compressibility of plywood had also some harmful effect induced by the comparatively low bending strength of the stringer flanges about the longitudinal axis. The flanges bent, as indicated in Fig. 5, with formation of plastic hinges at the sections A on the sides of the web, reducing the restraints at the top and bottom of the web from fixed to something approaching pinned and thereby decreasing its resistance to buckling. The calculations leading to the conclusion presented here are based on the compressibility tests of the plywood pads, the results of some of which are shown in the graphs of Fig. 6.

Plywood Tests

The plywood used in the tests was of the same kind as on the bridge. It was 3/4" thick and was made of 7 plies of Douglas Fir. At the time of testing it was air-dry with 7% moisture content. In all cases the plywood was compressed between the table of the testing machine and the fixed upper head which was in the form of a circle 10" in diameter. Some of the specimens

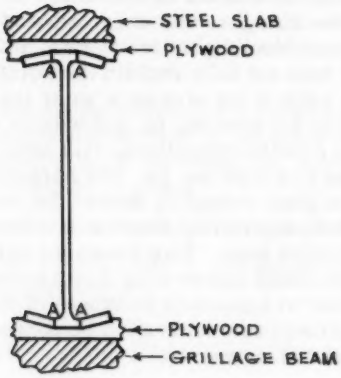

STEEL SLAB
PLYWOOD
A A
A A
PLYWOOD
GRILLAGE BEAM

FIG. 5

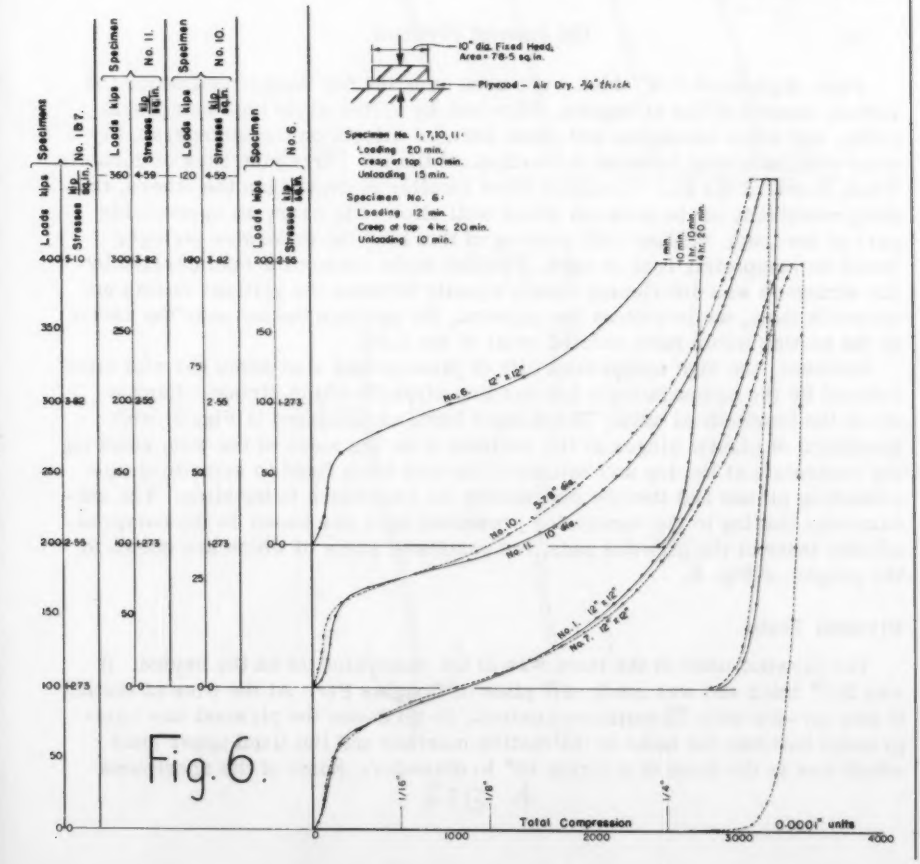

10" dia. Fixed Head, Area = 78·5 sq. in.
Plywood – Air Dry. ¾" thick.
Specimen No. 1, 7, 10, 11:
Loading 20 min.
Creep of top 10 min.
Unloading 15 min.
Specimen No. 6:
Loading 12 min.
Creep of top 4 hr. 20 min.
Unloading 10 min.
Specimens
Loads kips
Stresses kip/sq.in.
No. 1 & 7.
Specimen
No. 11.
Specimen
No. 10.
Specimen
No. 6.
No. 6. 12" x 12"
No. 10. 5·78" dia.
No. 11. 10" dia.
No. 1. 12" x 12"
No. 7. 12" x 12"
1 min.
10 min.
1 hr. 10 min.
4 hr. 20 min.
1/16"
1/8"
1/4"
Total Compression
0·0001" units

Fig. 6.

were squares 12" x 12", others were circles 10" in diameter and still others —smaller circles—5.78" in diameter having an area exactly 1/3rd of the area of the bigger circles. The specimens were loaded to a unit stress somewhat higher than the estimated stress of 2 - 2 1/4 kips/sq. in. experienced in the grillage; they were kept for a period of time under the full load and were then unloaded.

The stress-strain curves of the plywood were characterized by the following features:

1. Plotted to the same scale of unit stress, all curves were practically the same thus proving that the unit stress vs. unit strain relations did not depend on the areas loaded.

2. In the region up to a stress of 0.6 - 0.7 kips/sq. in. the curves were comparatively steep and straight with the corresponding modulus of elasticity of some 40 kips/sq. in. Between the stresses of approximately 0.7 and 1.5 kips/sq. in. the curves were flat with the modulus of elasticity reduced to as little as 4 kips/sq. in. Beyond the flat region the slope of the curves increased gradually to a modulus as great as 30 kips/sq. in.

3. The curves of unloading were very steep. There was virtually no rebound of deformation on removel of 85 - 90% of the load. A small rebound occurred on removal of the final remains of the load. This peculiarity of the stress-strain relation of plywood had an important bearing on the resistance of the web of the stringer to buckling, as is explained later.

4. When the load was kept constant the deformation continued to increase with time, in other words, the plywood was subject to creep, the rate of which, as is usual in creep tests, decreased quickly with time. Additional tests indicated that the amount of creep at different loads varied inversely with the slope of the stress-strain curve, in other words, the creep effects were greater on the flatter parts of the curve.

The stress-strain graphs described here made it possible to perform calculations pertaining to the load distribution on the stringers.

Formation of Plastic Hinges in Flanges of Stringers

Fig. 7 shows the deformations of the stringer flange and the plywood pad in

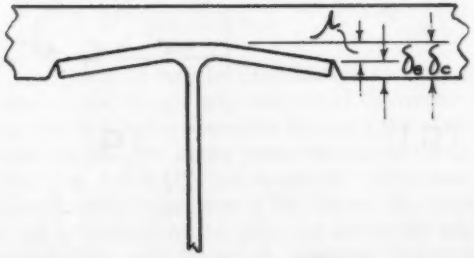


FIG. 7

transmitting the load from the column base to the stringers. The edges of the flange deflect an amount Δ in relation to the centre, and the plywood pad compresses the amounts δ_e at the edges and δ_c at the centre, since the heavy steel slab resting on the plywood pad virtually cannot bend.

Evidently, $$\Delta = \delta_c - \delta_e \qquad (1)$$

The distribution of unit pressure between the plywood and the flange is likely to be of the type shown in Fig. 8(a) with the edge pressure p_e being smaller than the centre pressure p_c on account of a smaller compression of plywood at the edges compared to the centre. In an extreme case the pressure at the edge may be zero as shown in Figs. 8(b) and 8(c).

That a uniform distribution of pressure in the grillage in accordance with Fig. 8(d) was impossible, becomes clear from the following calculation:

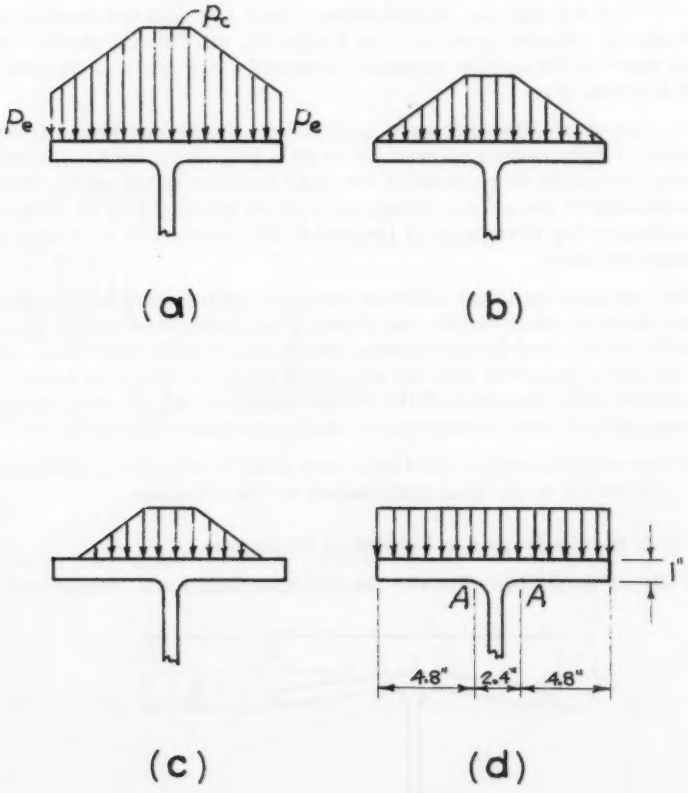


FIG. 8

Column load per stringer at the time of failure: 675 kips.
Length of base slab along the stringer: 42"
Load per inch of length of one stringer: $\frac{675}{42} = 16.0$ kips/inch.

Referring to Fig. 8(d): load per sq. in.: $p = \frac{16.0}{12} = 1.32$ kips/sq. in.

Considering one inch of the length of the flange, the bending moment at the beginning of fillet, point A, is

$$M_A = 1/2\ (1.32)\ (4.8)^2 = 15.2 \text{ kip-inches/inch.}$$

With $I/_C = \frac{1}{6}\ (1)\ (1)^2 = \frac{1}{6}$ inch^3/inch, the maximum bending stress f = 15.2 (6) = 91.2 kips/in.^{2}. This stress is so great that it calls for development of plastic hinges at A.

Using the plastic section modulus $I/_C = 1.5(\frac{1}{6}) = \frac{1}{4}$ inch^3/inch instead of the elastic, in the formula for the bending stress, the latter comes to 15.2(4) = 60.8 kips/sq. in. which is again much greater than the yield stress. This calculation demonstrates the impossibility of a uniform distribution of pressure over the flange in accordance with Fig. 8(d). The pressure then must be distributed in the manner of one of the figures 8(a), (b) or (c).

With the stresses at the edges of the flanges thus reduced, the moment at the points A must decrease and an additional calculation is needed in order to demonstrate that the plastic condition in the flanges is unavoidable.

Assume tentatively, by way of demonstration "ad absurdum" the existence of the limiting elastic stress condition in the flange, with the extreme bending stress equal to the yield stress 33 kips/sq. in., and find the deflection Δ of the edge of the flange in relation to the centre, as in a fixed-ended cantilever beam of 4.8" length (Fig. 9).

Using a uniform pressure distribution (Fig. 9(a)), by the moment-area theorem, remembering that $\frac{M}{EI} = \frac{f}{Ey}$,

$$\Delta = \frac{1}{3}\ (4.8)\ \frac{33}{(30{,}000)(0.5)}\ (\tfrac{3}{4})\ (4.8) = 0.0127"$$

Using a triangular pressure distribution (Fig. 9(b)

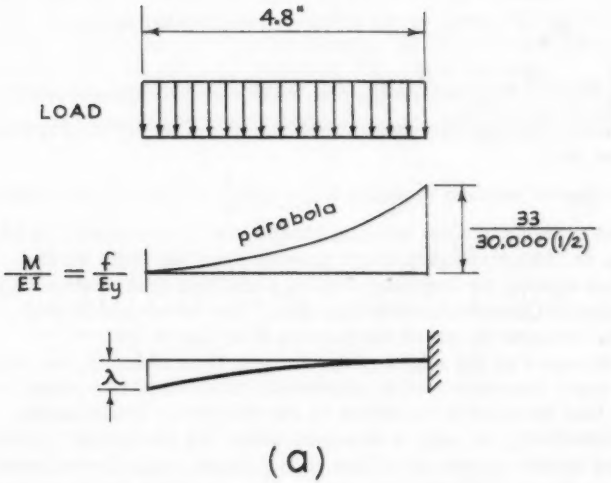
$$\Delta_1 = \frac{1}{4}(4.8)\ \frac{33}{(30{,}000)(0.5)}\ (4/_5)\ (4.8) = 0.0102"$$

Thus no matter how the load may be distributed over the flange an elastic deflection of the edge of the flange may only be of the order of 0.01".

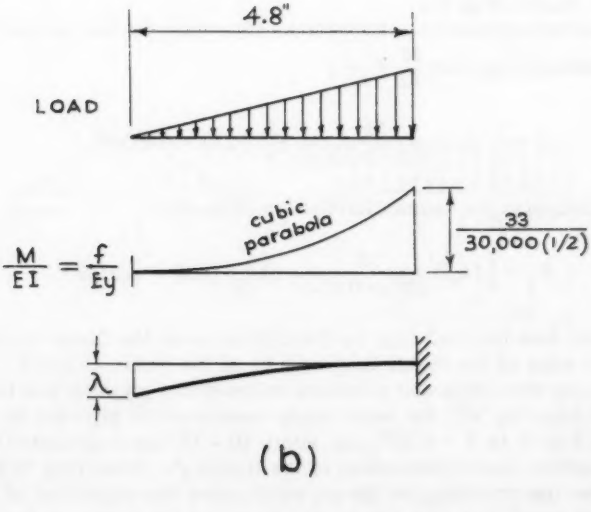
Remembering that the mean pressure between the plywood and the stringer flange is 1.32 kips/sq. in., the mean compression of the plywood is found by the graphs of Fig. 6 as $\delta = 0.12"$, i.e. some 10 - 12 times greater than the maximum possible elastic deflection of the flange Δ. Referring to Eq. (1) this means that the crushings of the plywood under the edges and at the centre must be nearly equal, but with the equal crushings the pressure between the plywood and the stringer flange must be uniform. This deduction, however, has been disproved. Then the assumption of the elastic stress condition in

flanges must be false too, and the flange must deform plastically.

The gist of this reasoning may be stated in this manner: the combination of high compressibility of plywood and of a very small maximum possible elastic deflection in the flange would make the pressure on the flange under the elastic conditions nearly uniform, which however is impossible in view of the low bending strength of the flange. The conclusion then is that the flanges bend inelastically forming plastic hinges, i.e. sharp bends, at the sections A,



(a)



(b)

FIG. 9

thus decreasing the compression of plywood at the edges and producing the pressure distribution in the manner of Figs. 8(a), (b) or (c).

It may be pointed out also that in the absence of the plywood pads, i.e. with the column base resting directly on the flanges, the pressure distribution would be still of the types of Fig. 8(b) or (c), but the bending of the flange would be elastic.

The presence of the elastic or plastic state of bending in the flange makes a great difference in the resistance of the stringer web to buckling, as is illustrated in Figs. 10(a) and (b). When the steel slab rests directly on the stringer and bending is elastic a slight clockwise tilting of the stringer, as a stage preliminary to buckling, brings into play a resisting counter clockwise moment through a tighter contact and a greater pressure between the slab and the beam on the left side of the flange and a looser contact on the right side bringing with it an elastic reduction of the pressure. When however plywood is interposed between the slab and the beam (Fig. 10(b)), whose flanges are bent inelastically, the resistance to a similar clockwise tilting is greatly reduced because a tighter contact on the left side brings no more pressure as the left side of the flange rotates about the plastic hinge, while a looser contact on the right side of the flange tends to remove nearly all pressure due to a virtual absence of an elastic rebound in the plywood, already referred to in the course of the description of the plywood tests. In order to continue to develop the normal load P and at the same time to offer a due moment resistance the flange must crush the plywood deeply and must turn much more in the direction of tilting than when the plywood is absent. This behaviour signifies a greatly reduced resistance to buckling in the presence of plywood.

It should be pointed out that the high compressibility of plywood can produce the undesirable effects discussed here only when combined with a low bending strength of the flange. A greater thickness of the flange would preclude the possibility of formation of plastic hinges and the reduction of buckling resistance. Again, the presence of effective diaphragms between the stringers would stabilize them and prevent their buckling even in spite of the formation of the plastic hinges in their flanges.

Imperfect Shape of Stringers

Rolled sections are never geometrically perfect and definite small tolerances are permitted in mill practice with regard to their deviations from the theoretical dimensions and shapes. Normally the effect of these imperfections on strength is minor and need not be specially provided for in design, but occasionally these irregularities give rise to dangerous situations, which must be recognized by proper provisions in the specifications. Buckling of the webs of the grillage stringers is one of these cases. The danger of this condition arises from the fact that specifications, such as AASHO, give only one column formula for the design of compression members and the imperfections visualized in this formula are relatively much smaller than the ones encountered in compression of the webs of the beams.

The use of an ordinary column formula in checking the buckling of the webs of stringers in grillages is apparently a recognized practice judging by the examples presented in the book by Shedd* who uses for this purpose the regular AREA and AISC column formulae. This practice in the opinion of the author is unsafe.

In order to study experimentally the buckling strength of the beam web the writer tested two pieces of 36" WF at 160# - 12" long, cut from crops left from fabrication of the grillage stringers involved in the failure of the bridge. The two specimens were nearly identical having been made from adjacent

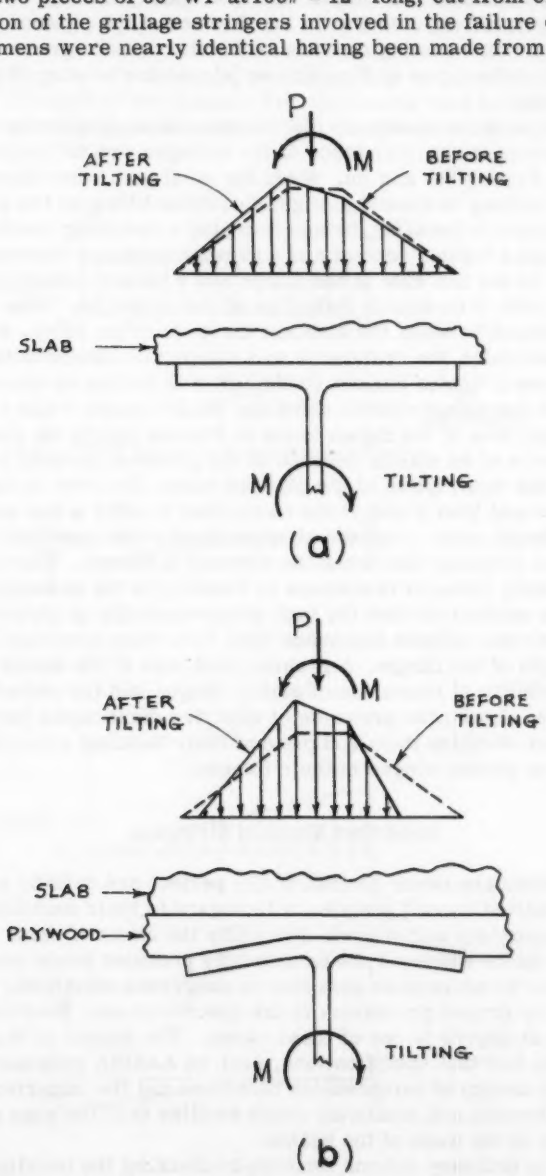


FIG. 10

parts of the same length. The tests were performed in the universal testing machine with the upper head fixed, so that both the top and the bottom flanges of the beam behaved as fixed in direction and position. This was not the same condition of end restraints as in the bridge grillage, but it was more convenient to reproduce in the laboratory and it served its purpose. The arrangement of the tests is shown in Figs. 11 and 12. The two tests differed only in the presence of the plywood pads interposed between the loading plates and the beam flanges in the set-up of Fig. 12. No such pads were present in the other test.

A significant feature of the test specimens was the presence of numerous imperfections.

1. The web of the beam was 3/16" off centre making the widths of the two halves of both flanges different by 3/8".
2. The shape of the cross-section was not truly rectangular so that while the top flange of the specimen, after being placed in the testing machine,

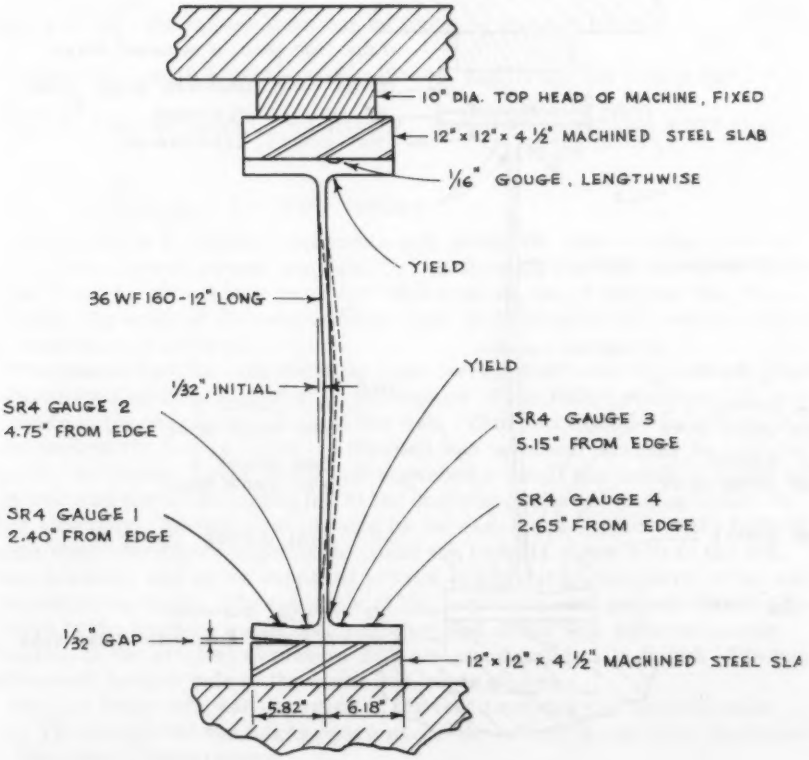


FIG. 11

was fully covered by the bearing plate the bottom flange had a gap under one half of it as great as 1/32" at the edge.

3. The top flange had a narrow depression or a gouge 1/16" deep, running lengthwise and located just off centre.

4. The web had a curvature whose centre rise was approximately 1/32".

The imperfections (1) and (2) were within the tolerances permitted by the usual practice.** The permissible limits of the curvature of the web and of the depression in the flange are not known to the writer but the fact that the stringers passed the bridge inspection leads him to believe that all irregularities were within the allowable limits.

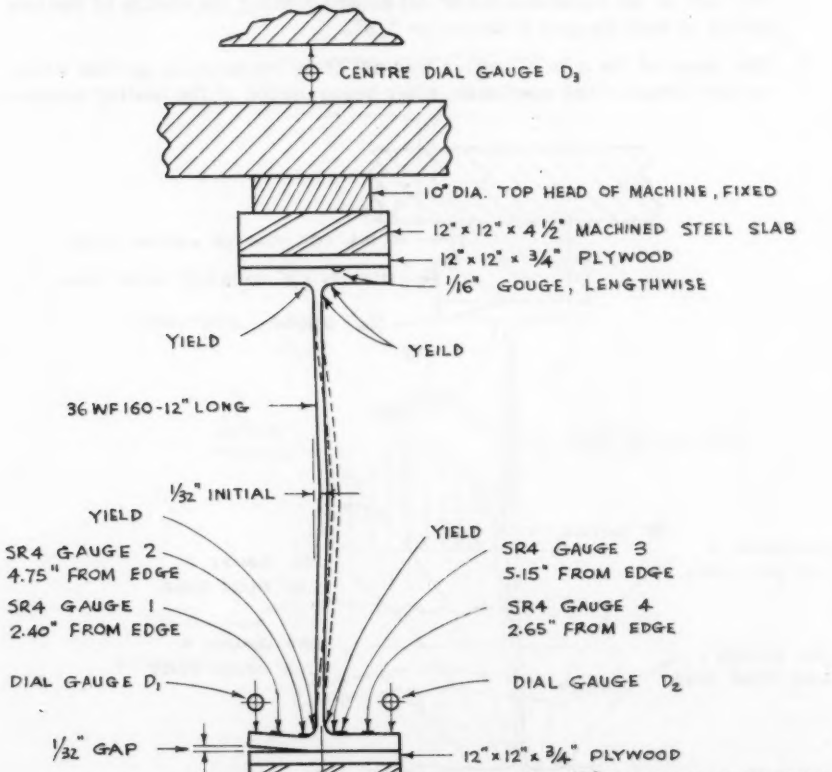


FIG. 12

** Manual of the American Institute of Steel Construction, 1948, p. 64.

The following values of the maximum loads and the corresponding unit stresses were observed:

Specimen with Plywood:

Maximum load 139 kips, unit stress, assuming uniform compression, $\frac{139}{12(0.653)} = 17.73$ kips/sq. in.

Specimen without Plywood:

Maximum load 143 kips, unit stress assuming uniform compression $\frac{143}{12(0.653)} = 18.23$ kips/sq. in.

By the way, as has already been stated, the actual stress in the stringer web of the grillage at failure, with flanges less firmly restrained than in the test, was found to be 17.2 kips/sq. in.

Considering the test as a pure compression of a fixed-ended column of the length $L = 32\frac{1}{4}$" the failure load may be found by Euler's formula $P_{cr} = \frac{\pi^2 EI}{\ell 2}$, using for ℓ the length $1/2(32\frac{1}{4}) = 16\frac{1}{8}$" (Shedd, p. 87), and for I the value $I = \frac{12(0.653)^3}{12} = 0.2787$ inch^4. This gives $P_{cr} = \frac{\pi^2(30{,}000)(0.2787)}{(16\frac{1}{8})^2} = 317$ kips

and $f_{cr} = \frac{317}{(12)(0.653)} = 40.5$ kips/sq. in.

Since Euler's formula is applicable only within the elastic range and the value of the critical stress computed by this formula exceeds the lower yield stress found by experiment to be $f_y = 33.9$ kips/sq. in., it follows that, theoretically, the critical stress should be close to 33 kips/sq. in. and the critical load close to 258 kips.

The reason that the true buckling loads were found to be only half as great as calculated must be sought in imperfections of the rolled sections and particularly in the initial curvature of the web. This conclusion was strongly suggested by the test in which the plywood was not used. As may be seen in Fig. 11, the bottom flange of the specimen had a small gap under it on the left side and was therefore supported at the commencement of testing under its right side only. The moment created by the one-sided support of the bottom flange might have been expected to cause the bending of the web to the left. This, however, had never happened in view of the initial curvature of the web directed to the right. The influence of the curvature was greater than the influence of the moment at the bottom flange and so the web deflected in the direction of the original curvature until the specimen finally failed. The gap underneath the left side of the beam had never closed.

Similar behaviour was observed in the test involving the plywood pads (Fig. 12) except that the empty space under the bottom flange soon disappeared as the plywood compressed.

Electrical resistance S.R.4 strain gauges were used on the inside of the bottom flanges. In the test with plywood the gauges registered high stresses indicating formation of plastic hinges next to the web as predicted by the theoretical considerations. Peeling of the shop paint also indicated formation of plastic hinges at the two extremities of the web (see Fig. 12) when the latter

began to buckle. In the test without the plywood, peeling of the paint was also noticed at the approach of buckling failure at two locations, namely, at the bottom of the web and on the right hand side of the top flange (See Fig. 11) where the latter was weakened by a deep depression.

These results thus proved definitely the weakening effect of the beam imperfections, even though these imperfections were probably not unusual. The tests also confirmed the influence of the compression of plywood on formation of plastic hinges in flanges, but they were non-conclusive in demonstrating the weakening effect of plywood on the buckling of the webs of the stringers, since the two observed values of the buckling load, 139 kips with plywood and 143 kips without it, were not sufficiently far apart. Apparently the effect of the imperfections on the buckling load was so great as to nearly conceal the unfavourable influence of the compressibility of the plywood.

Difference of Depths of Stringers

Evidence was found suggesting a difference in depth of the four stringers forming the grillage, with the extreme northerly stringer being 1/4" undersize. Although the plywood pads were beneficial in this connection they could not equalize the loads perfectly and the second northerly stringer must have carried an excess load estimated at 10%. Under the conditions resulting in failure, overstress of this magnitude probably did have some significance but normally it should be covered by the usual factor of safety inherent in the allowable stresses.

Column Formulae

The results of the compression tests of the beam sections showed the important effect on their strength of the imperfections of the shape, which might at first glance seem trivial. In the light of these findings it appears incorrect to design grillage beams in buckling by the same column formulae as are used for the compression members in general. The usual column formulae are intended primarily for the members of trusses and steel frames and although they provide for the presence of some imperfections and defects in shape, these imperfections are apparently less significant than the ones encountered in the beam sections subjected to buckling of the web. This may be demonstrated by applying the column formula to the tests just discussed:

$$\text{Using } \ell = \frac{1}{2}(32\tfrac{1}{4}) = 16\tfrac{1}{8} \text{ (Shedd, p. 87)}$$

$$\text{and } \quad r = \frac{0.653}{\sqrt{12}} = 0.191, \ \frac{\ell}{r} = 85.5$$

Then the allowable stress by AASHO, 1955 specification, with an addition of 33% allowance for erection conditions, comes to 16.8 kips/sq. in. This value is much too close to the failure stresses 17.73 and 18.23 kips/sq. in. observed in the tests. These results show that a new and more conservative column formula must be devised for the design of grillage webs in buckling in the absence of web stiffeners.

Attention must be drawn also to the inadequacy of the column formula of

CSA, 1952 specifications, not only for the design of grillages but for general design purposes. In the statement of this formula the length ℓ is defined as the unsupported length without specifying the conditions of restraint on the ends. Although the end conditions in compression members are often somewhat indeterminate, this is not always so, and there are situations in which the ends are pinned, as is recognized in the AASHO column formula and as is the case with webs of grillage beams subjected to buckling. Since the CSA column formula does not disqualify such members it should be safe in application to them, but actually it is not. This statement may be best demonstrated by the three curves (Fig. 13) representing the compression stress as a function of the slenderness ratio ℓ/r. The curves (1) and (2) show the allowable stresses, with an additional 33% allowance for erection, corresponding respectively to the column formulae of AASHO, 1955 and CSA, 1952 specifications. The limits of applicability of these formulae in terms of ℓ/r, are also indicated in the figure. The curve (3) represents the failure stress of pin-ended columns in accordance with Euler's formula. The comparison of these curves leads to the striking deduction that a pin-ended member, of slenderness ratio between 145 and 190, designed for erection conditions by the Canadian column formula possesses a factor of safety less than one, which signifies that the member so designed must fail. Thus the CSA column formula is grossly unsafe when applied to pin-ended members within the indicated limits of the slenderness ratio.

The allowable stresses prescribed by AASHO are everywhere some 10 - 25% lower than the ones given by CSA. Furthermore, the extreme limits of applicability of the AASHO formula are set in such a way as to make it safe everywhere, except of course for the design of the webs of grillage beams, as was demonstrated earlier.

FIG. 13

CONCLUSION

The following conclusions may be drawn from the study of the grillage failure leading to the collapse of the Second Narrows Bridge in Vancouver:

1. Compressibility of the plywood pads interposed between the column bases and the flanges of the grillage beams normally leads to sharp bending of the beam flanges about the longitudinal axis. The resultant formation of plastic hinges on the flanges may produce an unfavourable effect on the stability of the beam webs in buckling.

2. The seemingly minor imperfections in the shape of rolled sections of beams, particularly the curvature of the web, have a strong effect on the buckling strength of the web. The usual column formulae are not safe in the design of the webs of beams in buckling.

3. The column formula given by the Canadian specification CSA 1952 is unsafe for design of pin-ended members within certain limits of the slenderness ratio.

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CUT-OFF POINTS FOR COVER PLATES

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ABSTRACT

The coverplate at the bottom flange of a composite beam may be cut off short of the supports.

The exact computation of the theoretical length is tedious.

A method is presented which results in a chart from which the theoretical cut-off point can be determined with sufficient accuracy.

One of the operations in designing composite I-Beam stringers is the determination of the cut-off point for the cover plate which is welded to the bottom flange.

Most designers use approximate formulas.[1][2] These formulas will give results of varying accuracy depending on the loading conditions of the stringers. Some designers determine the moments and stresses stradling the cut-off point and by means of linear interpolation fix its exact location. This method is somewhat time consuming.

The following method results in a chart from which the cut-off point can be determined directly and with more accuracy than by the formulas mentioned above.

If only dead loads were acting on a stringer of constant cross-section the stress curve would follow curve "A" (Fig. 1) which is a parabola.

Equation;

$$x_1^2 = \frac{L^2}{4f} y_1 \qquad (1)$$

f is the stress at midspan.

The stress curve for standard truck loading (H or H-S trucks) will follow curve "B" (Fig. 1). The length of the straight portion "g" at the center of the

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span equals 4.67 feet for H-S truck loading for spans exceeding 33.8 feet. For other types of loading it will be necessary to determine the corresponding "g" values. The curved portion follows the envelope of maximum moments. A parabola closely approximates the actual curve.

Equation:
$$x_2^2 = \frac{(L-g)^2}{4f} y_2 \qquad (2)$$
$$x_2 = x_1 - \frac{g}{2}$$

For combined dead and live loading the stress curve would follow curve "C" (Fig. 1). Curve "C" depends on the ratio of the dead load to the live load stresses. Substituting

$$\frac{f_{LL}}{f_{DL}} = \alpha, \quad \frac{g}{L} = \beta, \quad \frac{x_1}{L} = b$$

f_{LL} is the stress at midspan due to live loads

f_{DL} is the stress at midspan due to dead loads

We can rewrite equations (1) and (2) as follows:

$$y_1 = b^2 \times 4 f_{DL} \qquad (1a)$$
$$y_2 = \frac{(x_1 - \frac{g}{2})^2}{(L-g)^2} \times 4 f_{LL}$$

$$y_2 = \frac{(b - \frac{\beta}{2})^2}{(1-\beta)^2} \times 4 \alpha f_{DL} \qquad (2a)$$

$$y = y_1 + y_2 = \frac{4 f_s}{1+\alpha} \left[b^2 + \alpha \frac{(b - \frac{\beta}{2})^2}{(1-\beta)^2} \right] \qquad (3)$$
$$f_s = f_{DL} + f_{LL} = f_{DL}(1+\alpha)$$

f_s is the combined stress due to dead and live loads at midspan. Equation (3) is the general equation for curve "C" between points "D" and "E" (Fig. 1).

Assuming that the span length "L" could vary from 36 to 94 feet, which is the approximate range for which rolled I-Beam stringers are used in composite bridge deck design, we find β varying from

$$\frac{4.67}{36} \cong .13 \quad \text{to} \quad \frac{4.67}{94} \cong .05$$

Using the above extreme values and using a value of b = .333 (most of the cover plate cut-offs are in this region) the following values are found for y in terms of f_s.

For $\alpha = .50$: $\beta = .13$, $y = .423 f_s$

$\beta = .05$, $y = .436 f_s$

y varies by 3%

For $\alpha = 1.50$: $\beta = .13$, $y = .406\, f_s$

$\beta = .05$, $y = .429\, f_s$

y varies by 5.6%

The above indicates that average values for β and α may be used in the general equation and sufficiently accurate results will be obtained.

From equation (3): b = .333, $\alpha = 1.00$, $\beta = .09$

$$y = .422\, f_s$$

The maximum possible error would be $\frac{16}{406}$ or 3.9% providing α stays within the limits of $.50 < \alpha < 1.50$. Most ratios of α in composite bridge deck design will fall within these limits. The general formula (3) thus reduces to

$$y = 2 f_s \left[b^2 + \left(\frac{b - .045}{.910} \right)^2 \right] \qquad (4)$$

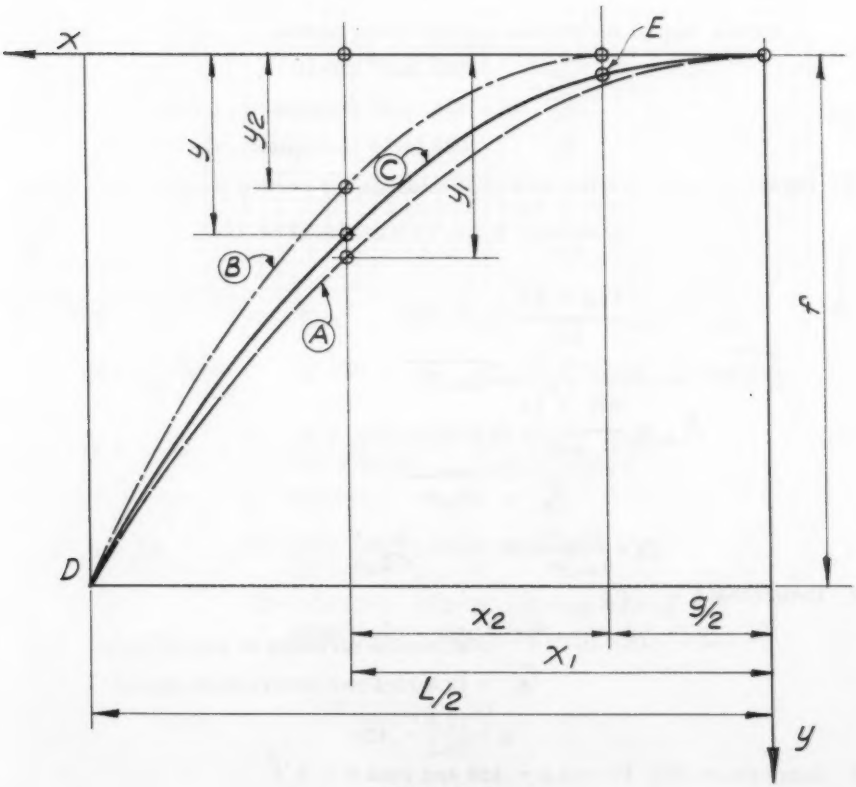


FIGURE 1

This equation is the basis for the chart shown in Fig. 2. A different chart would have to be developed for H truck loading, using proper g and β values.

Example:

Span = 67.08 feet, spacing of stringers = 7.42 feet

Moments at midspan;

Dead load carried by steel alone; w = 0.92 kips/foot

M_{DL}^{S} = 517 footkips

Dead load carried by composit section (Wearing surface);

w = .185 kips/foot

M_{DL}^{C} = 104 footkips

Live load, H20 - S16, Impact 26 %;

$$M_{LL} = \frac{7.42}{5 \times 2} \times 1.26 \times 932 = 871 \text{ footkips}$$

Section Moduli for section without cover plates;

S_c = 503 inch^3 (Steel)

S'_s = 623 inch^3 (Composite, n = 30)

S_c = 688 inch^3 (Composite, n = 10)

1. Determine bottom fiber stress at midspan for section without cover plate;

$$\frac{517 \times 12}{503} = 12.35 \text{ kips/square inch}$$

$$\frac{104 \times 12}{623} = 2.00 \quad "$$

$$f_{DL} = 14.35 \quad "$$

$$f_{LL} = \frac{871 \times 12}{688} = 15.15 \quad "$$

$$f_s = 29.50 \quad "$$

$$\alpha = \frac{15.15}{14.35} = 1.06 \quad \begin{matrix} > .5 \\ < 1.5 \end{matrix}$$

2. Determine a = $\frac{f_a}{f_s}$

f_s = allowable stresses at cut-off point

f_a = 13.5 kips per square inch (Spec.)

$$a = \frac{13.5}{29.5} = .458$$

3. Enter chart (Fig. 2) with a = .458 and read b = .374

x = .374 x 67.08 = 25.1 feet

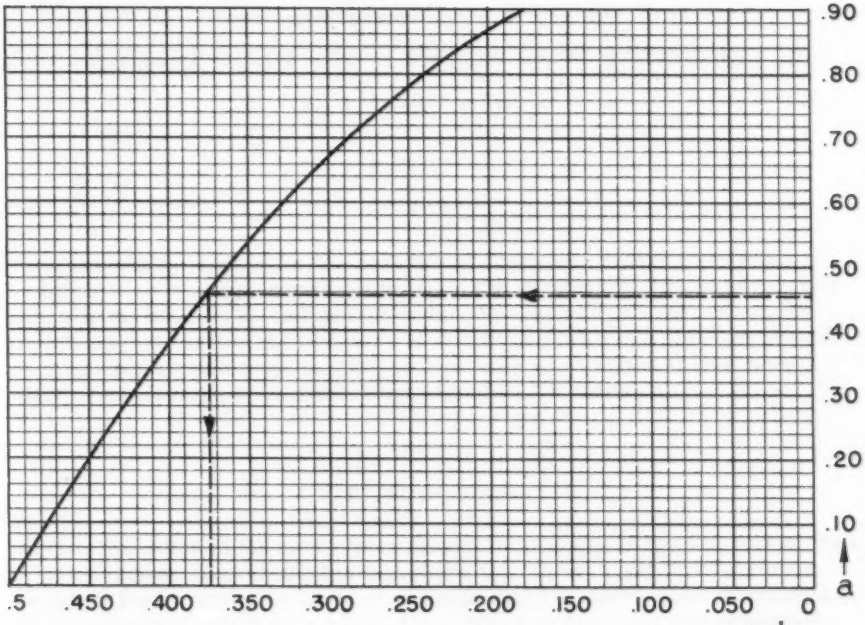
The theoretical length of cover plate is determined as

2 x 25.1 = 50.2 feet

REFERENCES

1. "Alpha Composite Construction Engineering Handbook," 3rd ed., Porete Manufacturing Co., North Arlington, N. J., 1953.

2. Viest, I. M., R. S. Fountain and R. C. Singleton: "Composite Construction in Steel and Concrete" McGraw Hill Book Co., N. Y., 1958.



$\alpha = \frac{fs\ L.L.}{fs\ D.L.}$ $1.5 > \alpha > .5$, H-S Truck Loading

$b = \frac{x}{L}$ x = distance from ℄ of span to theor. cut-off point.

L = span.

$a = \frac{fa}{fs}$ fa = allowable stress at cut-off point.

fs = bottom fibre stress at ℄ of span for section without C.℔.

FIGURE 2

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ANALYSIS OF CONTINUOUS TRUSSES BY CARRY-OVER MOMENTS

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SYNOPSIS

A general procedure for the analysis of continuous trusses subjected to stationary or moving loads is presented. The depth of the truss may be constant or variable. The outlined procedure is a successive approximation which may be carried out to a desired degree of accuracy. One starting value is computed at each support, and the continuity is established by means of carry-over factors only. Numerical examples are included.

INTRODUCTION

The extension of the carry-over moment procedure to the analysis of continuous trusses is presented in this paper.(1) The general three-moment equation for a continuous truss of variable depth is derived and introduced as a mathematical model from which the carry-over functions are developed.

The truss may be of constant or variable depth and the deformation of the structure may be caused by transverse loads, applied couples, displacements of support or inaccuracies in fabrication and erection.

The numerical procedure is similar in form to the moment distribution method(2) and to the balancing angle change method,(3,4,5) but it differs in the relaxation technique as well as in the concept of approach in that:

a) One assumed starting moment is computed at each support
b) One final moment is obtained at each support.
c) No distribution of unbalances is required during the relaxation procedure.
d) The relaxation procedure is carried out by means of carry-over factors only.
e) The procedure is self checking.

Note: Discussion open until May 1, 1960. To extend the closing date one month, a written request must be filed with the Executive Secretary, ASCE. Paper 2314 is part of the copyrighted Journal of the Structural Division, Proceedings of the American Society of Civil Engineers, Vol. 85, No. ST 10, December, 1959.

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This study is restricted to coplanar trusses and the customary assumptions of truss analysis are introduced. The sign convention of the three-moment equation is adopted.

The subsequent discussion is divided into six parts. The derivation of the general three-moment equation for a continuous truss of variable depth is shown in the first part of the paper.

The carry-over moment functions are defined in Parts 2 and 3. The application of moment coefficients, the numerical and algebraic control is discussed in Parts 4 and 5. In Part 6 the computation of influence lines for the moments at supports by means of numerical and algebraic carry-over moment procedure is presented. The meaning of enclosed tables is discussed in the Appendix.

1. Angular Functions

A continuous truss of variable depth subjected to a general system of loads is considered. The unyielding supports are denoted by 0, 1, 2, 3, . . . i, j, k, ... The exterior ends are simply supported and the bending moments at supports M_1, M_2, M_3, . . . M_i, M_j, M_k, . . . are selected as unknowns.

The panel (m) (any panel in the span $\overline{ij}$) given by the verticals through (m-1) and (m) contains bars designated by subscript (m). Similarly bars of panel (n) (any panel in the span $\overline{jk}$) are denoted by subscript (n) (Fig. 1).

The spans $\overline{ij}$ and $\overline{jk}$ are isolated into two free bodies (basic structures) (Fig. 2).

The total axial force in the bar m of the basic structure $\overline{ij}$

$$N_m = BN_m + M_i\alpha_m + M_j\beta_m \qquad (1a)$$

Where

BN_m = The axial force due to load

α_m = The axial force due to $M_i = 1$

β_m = The axial force due to $M_j = 1$.

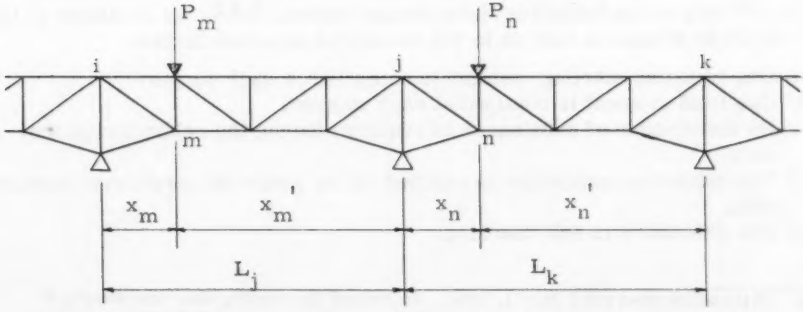


Fig. 1

Isolated Portion of a Continuous Truss

With similar notation, the axial force in the bar n of the basic structure $\overline{jk}$

$$N_n = BN_n + M_j\alpha_n + M_k\beta_n \quad . \tag{1b}$$

Where

BN_n = The axial force due to load

α_n = The axial force due to $M_j = 1$

β_n = The axial force due to $M_k = 1$

With notation

d_m, d_n = The length of the bar

A_m, A_n = The area of the cross-section of the bar

E = Modulus of elasticity of the bar

the axial flexibilities are:

$$\lambda_m = \frac{d_m}{A_m E} \quad \Big| \quad \lambda_n = \frac{d_n}{A_n E} \quad . \tag{2}$$

The strain energy of $\overline{ijk}$,

$$U_{ijk} = \frac{1}{2}\sum_{i}^{j} N_m^2 \lambda_m + \frac{1}{2}\sum_{j}^{k} N_n^2 \lambda_n \quad . \tag{3}$$

From the principle of minimum energy

$$\frac{\partial U_{ijk}}{\partial M_j} = \sum_{i}^{j} N_m \frac{\partial N_m}{\partial M_j} \lambda_m + \sum_{j}^{k} N_n \frac{\partial N_n}{\partial M_j} \lambda_n = 0 \quad . \tag{4a}$$

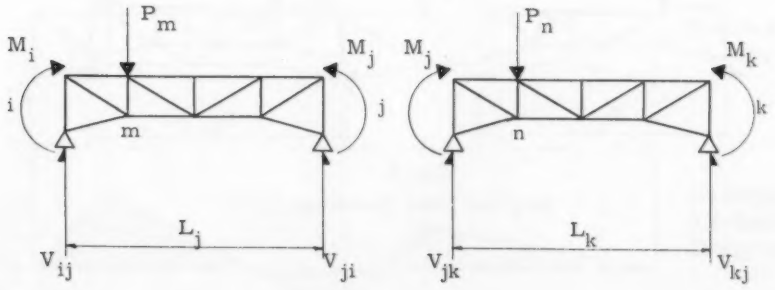


Fig. 2

Free Bodies $\overline{ij}$ and $\overline{jk}$

The equation (4a) in terms of Eq's (1a, b) and with new notation becomes a general three moment equation (4b).

$$G_{ij} M_i + (F_{ji} + F_{jk}) M_j + G_{kj} M_k = - \tau_{ji} - \tau_{jk} . \qquad (4b)$$

The meaning of symbols F's, G's, τ's and their physical interpretation follows.

(a) The angular flexibility F_{ji} (or F_{jk}) is the end slope of the basic structure $\overline{ij}$ (or $\overline{jk}$) at j due to a unit moment applied at that end (Fig. 3).

$$F_{ji} = \sum_{i}^{j} \beta_m^2 \lambda_m \quad \Big| \quad F_{jk} = \sum_{j}^{k} \alpha_n^2 \lambda_n . \qquad (5)$$

(b) The angular carry-over value G_{ij} (or G_{kj}) is the end slope of the basic structure $\overline{ij}$ (or $\overline{jk}$) at i (or k), due to a unit moment applied at the far end j (Fig. 3).

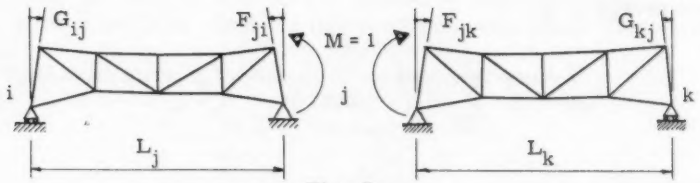


Fig. 3
Angular Flexibilities and Carry-Over Values

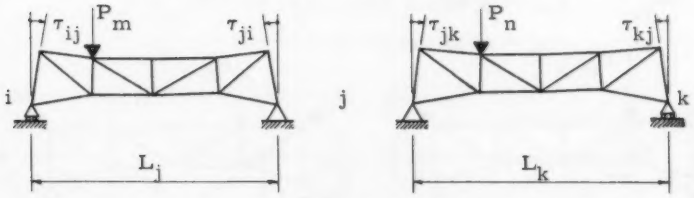


Fig. 4
Angular Load Functions

$$G_{ij} = \sum_{i}^{j} \alpha_m \beta_m \lambda_m \quad \Big| \quad G_{kj} = \sum_{j}^{k} \alpha_n \beta_n \lambda_n \ . \tag{6}$$

(c) The angular load function τ_{jk} (or $\tau_{\overline{jk}}$) is the end slope of the basic structure $\overline{ij}$ (of $\overline{jk}$) at j due to loads (Fig. 4).

$$\tau_{ji} = \sum_{i}^{j} BN_m \beta_m \lambda_m \quad \Big| \quad \tau_{jk} = \sum_{j}^{k} BN_n \alpha_n \lambda_n \ . \tag{7}$$

2. Moment Functions

With new equivalents

$$F_{ji} + F_{jk} = \Sigma F_j \tag{8}$$

$$-\frac{G_{ij}}{\Sigma F_j} = r_{ij} \quad \Big| \quad -\frac{G_{kj}}{\Sigma F_j} = r_{kj} \tag{9}$$

$$-\frac{\tau_{ji} + \tau_{jk}}{\Sigma F_j} = -\frac{\Sigma \tau_j}{\Sigma F_j} = m_j \tag{10}$$

the three moment equation (4b) simplifies to:

$$M_j = r_{ij} M_i + m_j + r_{kj} M_k \ . \tag{11}$$

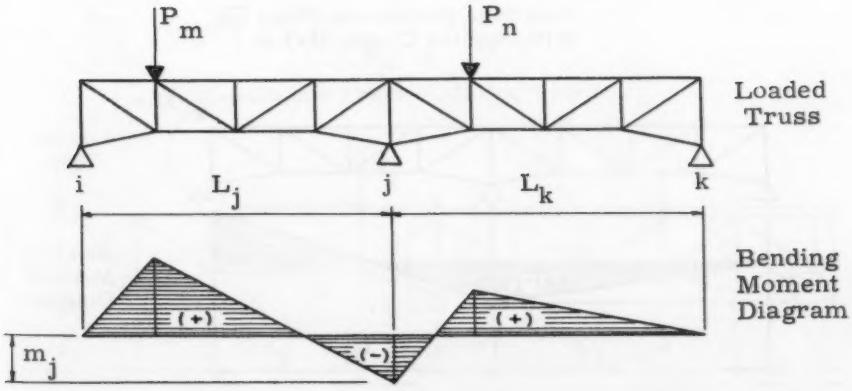


Fig. 5

Two Span Continuous Truss $\overline{ijk}$

The formal identity of Eq. (1) with the Eq. (11) of writer's recent paper[1] is apparent. The equivalents of this equation are again the starting moment m_j, the carry-over factors r_{ij}, r_{kj} and the redundant moments.

(a) The starting moment m_j is the bending moment at j, due to loads, if the moments M_i and M_k are equal to zero (Fig. 5).

(b) The carry-over factor r_{ij} is the bending moment at j, due to a unit moment at k is equal to zero (Fig. 6).

(c) The carry-over factor r_{kj} is the bending moment at j, due to a unit moment at k, if the moment at i is equal to zero (Fig. 7).

3. Special Conditions

Two special conditions may occur in the analysis of continuous trusses:

(a) Fixed end (Fig. 8)
(b) Overhanging end (Fig. 9).

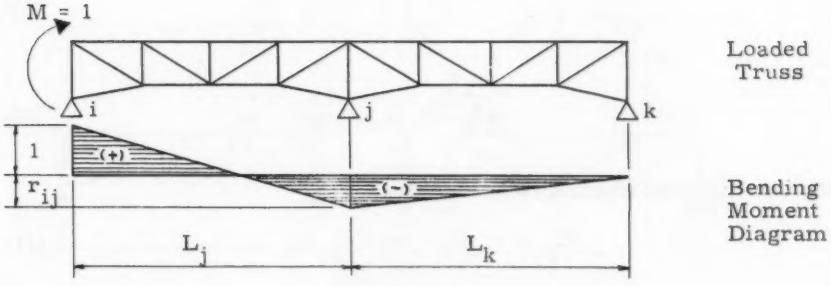


Fig. 6

Two Span Continuous Truss $\overline{ijk}$
With Applied Couple M=1 at i

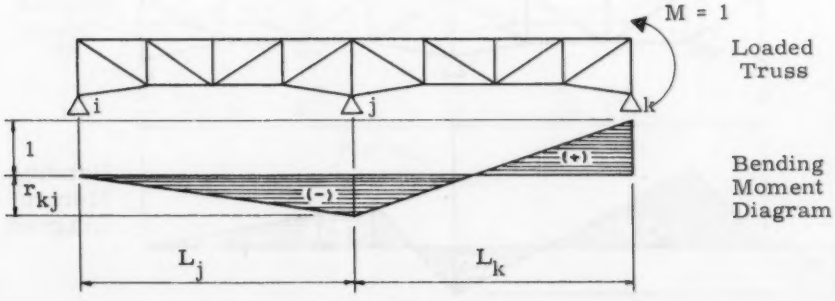


Fig. 7

Two Span Continuous Truss $\overline{ijk}$
With Applied Couple M=1 at k

If the end 0 of a continuous truss is fixed, the three-moment equation (11) reduces to:

$$M_0 = m_0 + r_{10} M_1 \tag{12}$$

where the starting moment (Eq. 10)

$$m_0 = - \frac{\tau_{01}}{F_{01}} \tag{13}$$

and the carry-over factor (Eq. 9)

$$r_{10} = - \frac{G_{10}}{F_{01}} \tag{14}$$

If the end 0 is free and the span L_1 overhangs the support 1, the starting moment at 1 is computed from statics and the carry-over factor

$$r_{21} = 0 \quad . \tag{15}$$

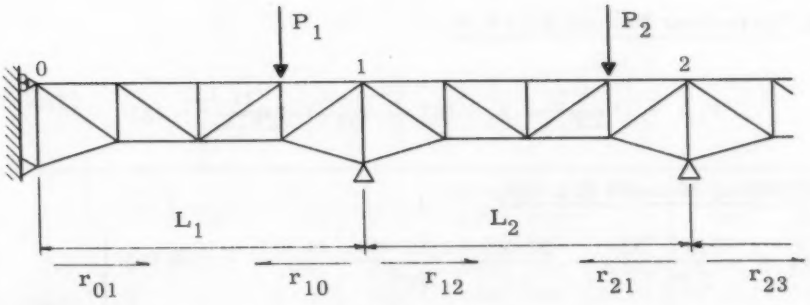


Fig. 8
Continuous Truss-Left End Fixed

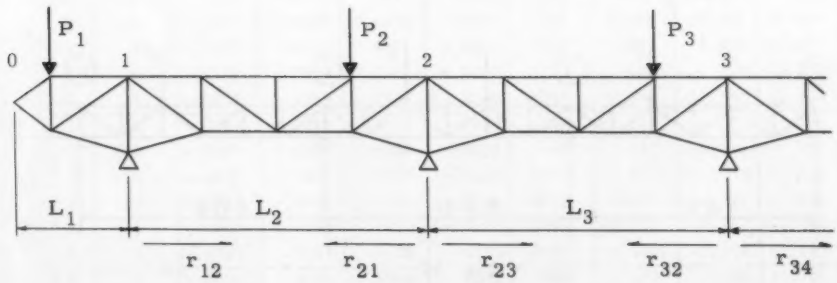


Fig. 9
Continuous Truss - Overhanging Left End

4. Numerical Procedure

Once the basic functions are defined (Eq's 9, 10, 12, 13, 14, 15), their application to the analysis of continuous trusses subjected to stationary loads may easily be shown. Two examples are introduced to illustrate the numerical procedure. Because the areas of all members are assumed to be the same, relative values of axial flexibilities are introduced.

$$\lambda_m = d_m \ .$$

All values are given in feet, kips, or kip-feet.

Example 1. A three span continuous truss of constant depth, loaded as shown (Fig. 10) is analyzed. The real structure $\overline{03}$ is resolved into three basic structures $\overline{01}$, $\overline{12}$ and $\overline{23}$. The computation of the elastic constants is demonstrated in Table 1.

(a) Relative Angular Flexibilities (Eq's 5, 8):

$$F_{10} = F_{12} = F_{21} = F_{23} = .9784$$
$$\Sigma F_1 = \Sigma F_2 = 1.9568 \ . \qquad \text{(Table 1)}$$

(b) Carry-Over Factors (Eq's 6, 9):

$$r_{12} = -\frac{G_{12}}{\Sigma F_2} = -.182 \qquad r_{21} = -\frac{G_{21}}{\Sigma F_1} = -.182. \qquad \text{(Table 1)}$$

(c) Starting Moments (Eq. 10):

$$m_1 = -\frac{\tau_{10} + \tau_{12}}{\Sigma F_1} = -\frac{23.575 + 2(19.089 + 23.575)}{1.9568} = -55.654$$
$$m_2 = -\frac{\tau_{21} + \tau_{23}}{\Sigma F_2} = -\frac{2(19.089 + 23.575) + 19.089}{1.9568} = -53.361. \qquad \text{(Table 1)}$$

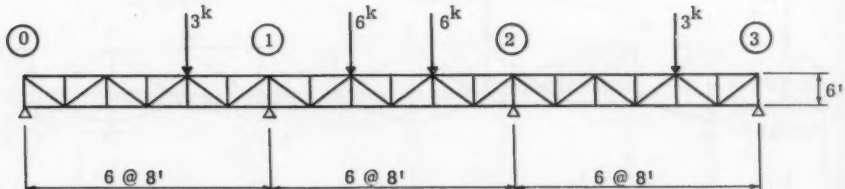
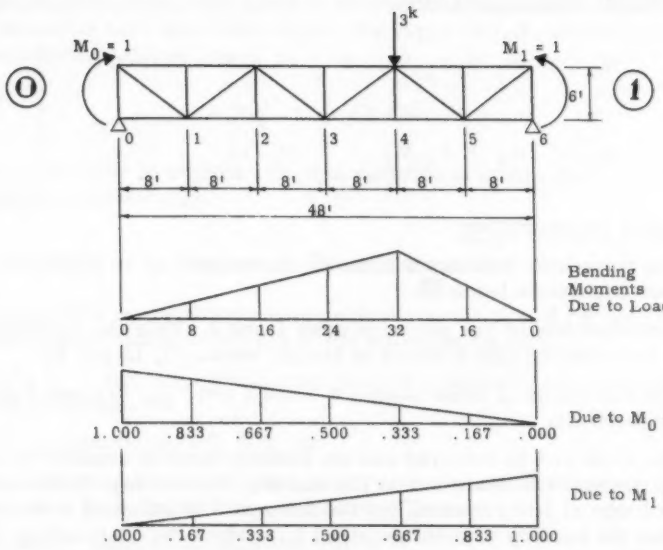


Fig. 10
Three Span Continuous Truss
of Constant Depth

Example 1 Basic Structure $\overline{01}$ Table 1



m		λ_m	α_m	β_m	$\alpha_m^2\lambda_m$	$\alpha_m\beta_m\lambda_m$	$\beta_m^2\lambda_m$	BN_m	$BN_m\alpha_m\lambda_m$	$BN_m\beta_m\lambda_m$
Bottom	1	8	+.1667	.0000	.2222	.0000	.0000	.000	.000	.000
	2	8	+.1111	+.0556	.0987	+.0494	.0247	+2.667	+2.370	+1.185
	3	8	+.1111	+.0556	.0987	+.0494	.0247	+2.667	+2.370	+1.185
	4	8	+.0556	+.1111	.0247	+.0494	.0987	+5.333	+2.370	+4.740
	5	8	+.0556	+.1111	.0247	+.0494	.0987	+5.333	+2.370	+4.740
	6	8	.0000	+.1667	.0000	.0000	.2222	.000	.000	.000
Top	1	8	-.1389	-.0278	.1543	+.0309	.0062	-1.333	+1.481	+.296
	2	8	-.1389	-.0278	.1543	+.0309	.0062	-1.333	+1.481	+.296
	3	8	-.0833	-.0833	.0556	+.0556	.0556	-4.000	+2.667	+2.667
	4	8	-.0833	-.0833	.0556	+.0556	.0556	-4.000	+2.667	+2.667
	5	8	-.0278	-.1389	.0062	+.0309	.1543	-2.667	+.593	+2.963
	6	8	-.0278	-.1389	.0062	+.0309	.1543	-2.667	+.593	+2.963
Diagonal	1	10	-.0347	+.0347	.0120	-.0120	.0120	+1.667	-.578	+.578
	2	10	+.0347	-.0347	.0120	-.0120	.0120	-1.667	-.578	+.578
	3	10	-.0347	+.0347	.0120	-.0120	.0120	+1.667	-.578	+.578
	4	10	+.0347	-.0347	.0120	-.0120	.0120	-1.667	-.578	+.578
	5	10	-.0347	+.0347	.0120	-.0120	.0120	-3.333	+1.157	-1.157
	6	10	+.0347	-.0347	.0120	-.0120	.0120	+3.333	+1.157	-1.157
Vert.	0	6	+.0208	-.0208	.0026	-.0026	.0026	-1.000	-.125	+.125
	6	6	-.0208	+.0208	.0026	-.0026	.0026	-2.000	+.250	-.250
Total					.9784	.3552	.9784		19.089	23.575

(d) Carry-Over Procedure (Table 2):

The carry-over procedure is performed in Table 2 by means of carry-over factors. Each final moment is equal to the sum of all values in the respective column.

(e) Numerical Control (Eq. 11):

$$M_1 = m_1 + r_{21}M_2 = -47.522$$

$$M_2 = m_2 + r_{12}M_1 = -44.712$$

(Table 2)

(f) Physical Interpretation:

1. The three span continuous truss $\overline{03}$ is replaced by an equivalent three span continuous beam $\overline{03}$.
2. Fictitious hinges are placed at point 1 and 2. Thus the continuous beam is transformed into a series of simple beams $\overline{01}$, $\overline{12}$ and $\overline{23}$.
3. The end slopes of these beams at 1 and 2 are τ_{10}, τ_{12} and τ_{21}, τ_{23}, respectively.
4. The hinge at 1 is removed and the bending moment required to produce the continuity at that point is the starting moment m_1. Independently, the hinge at 2 is removed, but the hinge at 1 is assumed to be preserved. Then the bending moment required to produce the continuity at 2 is the starting moment m_2.
5. Simultaneously, the starting moment m_1, now acting at the hinge 1, carries-over to 2 a carry-over moment $r_{12}m_1$; and the starting

Example 1 Carry-Over Table Table 2

Supports	1	2
Carry-Over Factors	- .182 →	← - .182
Starting Moments	-55.654	-53.361
1. C.O.M.	9.712 ↗	↖ 10.129
2. C.O.M.	- 1.843 ↗	↖ - 1.768
3. C.O.M.	.322 ↗	↖ .335
4. C.O.M.	- .061 ↗	↖ - .059
Final Moments	-47.524	-44.724

moment m_2, acting at the hinge 2, develops a carry-over moment $r_{21}m_2$ at 1. Because of the nature of the carry-over factors, the carry-over moment becomes at the same time a new starting moment.

6. The step (5) is repeated in a number of cycles until the continuity of the elastic curve is established.

The physical interpretation leads to an alternate procedure. After the starting moments have been determined, the larger m_1 is carried-over to 2 and added algebraically to m_2.

$$m_2^* = -43.232$$

when the carry-over is started with this modified moment, the numerical labor reduces considerably.

5. Alternate Numerical Procedure

The application of the alternate numerical procedure and the influence of special end conditions is presented in the following example.

Example 2. A five span continuous truss of constant depth as shown in Table 3 is analyzed.

(a) Elastic Constants (Eq's 5, 6, 7, 8):

The relative angular flexibilities, carry-over values and angular load functions taken from Table 1 are recorded in Table 3.

(b) Carry-Over Factors (Eq's. 9, 14, 15):

$r_{12} = -.182$	$r_{21} = 0$
$r_{23} = -.182$	$r_{32} = -.182$
$r_{34} = -.182$	$r_{43} = -.182$
$r_{45} = -.363$	$r_{54} = -.182$

(c) Starting Moments (Eq. 10, 13):

$m_1 = -16.000$	$m_2 = -9.755$
$m_3 = -12.048$	$m_4 = -9.755$
$m_5 = -24.095$	

(d) Carry-Over Procedure in Modified Form (Table 4):

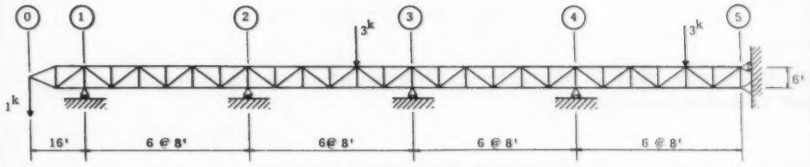
The starting moment at 1 and 3 is carried over to 2 and added algebraically to m_2. Thus the modified starting moment at 2,

$$m_2^* = -4.650 \quad .$$

Similarly, $m_4^* = -3.177$.

When these two modified starting moments are known, the carry-over procedure is carried on to a desired degree of accuracy. The final moments, equal to the sum of all values in the respective columns, are recorded in the same table.

Example 2 — Five Span Continuous Truss — Table 3



$F_{21} = .9784$	$\Sigma F_2 = 1.9568$	$G_{12} = G_{21} = .3552$	$\tau_{21} = \tau_{34} = \tau_{43} = 0$
$F_{23} = F_{32} = .9784$	$\Sigma F_3 = 1.9568$	$G_{23} = G_{32} = .3552$	$\tau_{23} = \tau_{45} = 19.089$
$F_{34} = F_{43} = .9784$	$\Sigma F_4 = 1.9568$	$G_{34} = G_{43} = .3552$	$\tau_{32} = \tau_{54} = 23.575$
$F_{45} = F_{54} = .9784$	$\Sigma F_5 = .9784$	$G_{45} = G_{54} = .3552$	

Example 2 — Carry - Over Table — Table 4

Supports	1	2	3	4	5
Carry-Over Factors	→ -.182	← 0 → -.182	← -.182 → -.182	← -.182 → -.363	← -.182
Starting Moments	-16.000	-9.755	-12.048	-9.755	-24.095
Modified		+2.912		+2.193	
Starting Moments		+2.193		+4.385	
1. C.O.M.			+.846		+1.153
			+.578		
2. C.O.M.				-.259	
		-.259		-.210	
3. C.O.M.			+.047		+.170
			+.085		
Total	-16.000	-4.909	-10.492	-3.646	-22.772

(e) Numerical Control (Eq. 11):

$M_2 = r_{12}M_1 + m_2 + r_{32}M_3 = -4.933$ | $M_3 = r_{23}M_2 + m_3 + r_{43}M_4 = -10.491$

$M_4 = r_{34}M_3 + m_4 + r_{54}M_5 = -3.700$ | $M_5 = r_{45}M_4 + m_5 = -22.772$

6. Moment Coefficients

If various load conditions or moving loads must be considered, the numerical labor can be reduced by using the method of moment coefficients in numerical or algebraic form. The derivation of the algebraic coefficients follows.

If the carry-over moment procedure is used to compute the moments over supports in three and four span continuous trusses and the procedure is expanded algebraically, each moment consists of a series that is infinite, convergent, and geometric (hereafter denoted as a basic series).

To illustrate the computation of final moments by means of basic series, a three span continuous truss of variable depth is considered (Fig. 11). The carry-over factors are denoted by B and C and the starting moment by $m_1 = 1$. The algebraic carry-over procedure follows in Table 5. The series formed by carry-over moments in each column is a basic series. When all terms in the first column of Table 5 are summed, the final moment due to m_1 becomes:

$$M_1 = 1 + (BC)^1 + (BC)^2 + \ldots = \frac{1}{X_{12}}$$

where

$$X_{12} = 1 - BC$$

Similarly

$$M_2 = \frac{B}{X_{12}}$$

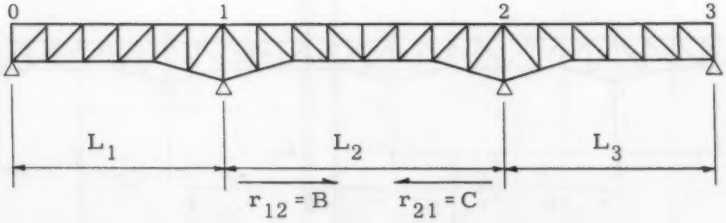


Fig. 11
Three Span Continuous Truss

Final moments due to m_1 and m_2 are recorded in Table 6.

In the case of a four span continuous truss of variable depth (Fig. 12), the final moments are formed by basic series whose denominator

$$Y_{13} = 1 - BC - DE \ .$$

All final moments similarly derived are recorded in Table 7.

Table 5 - Three Span Continuous Truss
Algebraic Carry-Over Table

Supports	1	2
Carry-Over Factors	$\overrightarrow{B}$	$\overleftarrow{C}$
Starting Moments	1	0
1. C.O.M.		B
2. C.O.M.	BC	
3. C.O.M.		B^2C
4. C.O.M.	B^2C^2	
5. C.O.M.		B^3C^2
⋮	⋮	⋮
Final Moments	$\sum$	$\sum$

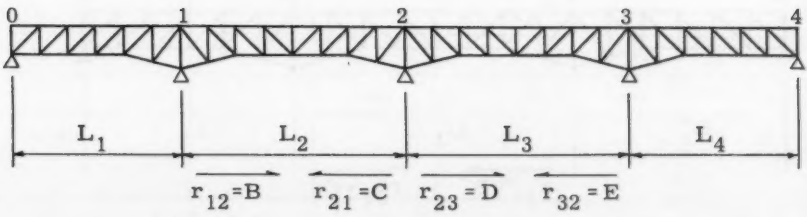


Fig. 12
Four Span Continuous Truss

The identity of Tables 6 and 7 in this paper with Tables C and D presented in writer's previous paper[1] is well apparent. The investigations in this paper are limited to three and four span continuous trusses. Extension of the philosophy of the algebraic carry-over procedure to a larger number of spans is possible. Moment tables for five, six, seven and eight span continuous girders of variable depth have been developed by Havner and writer.[18] These tables are well suitable for the analysis of five, six, seven and eight span continuous trusses of constant or variable depth and may be applied without modification.

7. Influence Line

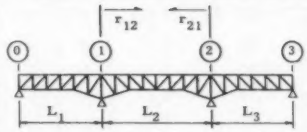
The application of the numerical and algebraic carry-over moment procedure to the analysis of continuous trusses of variable depth is shown in the following example. The geometry of the truss is simplified and the number of panels is reduced to fit in the limited scope of this paper. For the same reason the cross-sectional areas of all bars are assumed to be equal to unity. All values are given in feet, kips and kip-feet.

Example. Influence lines for the bending moments M_1, M_2, M_3, in a symmetrical, four span truss (Fig. 13) are required.

The real structure $\overline{04}$ is resolved into four simple trusses ($\overline{01}$, $\overline{12}$, $\overline{23}$, $\overline{34}$). Because of symmetry only the elastic constants of $\overline{01}$ and $\overline{12}$ are computed. The functions of basic structures (Table 8, 9) and of conjugate structures (Table 10, 11) are calculated by means of formulas assembled in Tables 12, 13. Relative values in terms of

$$\lambda_m = d_m$$

Three Span Truss — Table 6

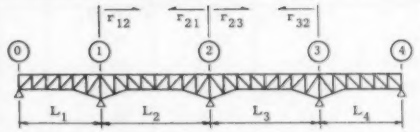


$X_{12} = 1 - r_{12}r_{21}$

Final Moments

	m_1	m_2
M_1	$\frac{1}{X_{12}}$	$\frac{r_{21}}{X_{12}}$
M_2	$\frac{r_{12}}{X_{12}}$	$\frac{1}{X_{12}}$

Four Span Truss — Table 7



$Y_{13} = 1 - r_{12}r_{21} - r_{23}r_{32}$

Final Moments

	m_1	m_2	m_3
M_1	$1 + \frac{r_{12}r_{21}}{Y_{13}}$	$\frac{r_{21}}{Y_{13}}$	$\frac{r_{21}r_{32}}{Y_{13}}$
M_2	$\frac{r_{12}}{Y_{13}}$	$\frac{1}{Y_{13}}$	$\frac{r_{32}}{Y_{13}}$
M_3	$\frac{r_{12}r_{23}}{Y_{13}}$	$\frac{r_{23}}{Y_{13}}$	$1 + \frac{r_{23}r_{32}}{Y_{13}}$

are used in all calculations. The lengths of members and lever arms are scaled from the drawings.

(a) Relative Angular Flexibilities:

$$F_{10} = F_{34} = +.2797 \qquad \text{(Table 10)}$$

$$F_{12} = F_{21} = F_{23} = F_{32} = +.3904 \qquad \text{(Table 11)}$$

$$\Sigma F_1 = \Sigma F_3 = +.6701 \,\big|\, \Sigma F_2 = +.7808 \,. \qquad \text{(Eq. 8)}$$

(b) Carry-Over Factors:

$$G_{12} = G_{21} = G_{23} = G_{32} = +.2135 \qquad \text{(Table 11)}$$

$$r_{12} = r_{32} = -\frac{.2135}{.7808} = -.273$$

$$r_{21} = r_{23} = -\frac{.2135}{.6701} = -.319 \,. \qquad \text{(Eq. 9)}$$

(c) Numerical Coefficients:

The moment coefficients due to $m_1 = 1$ and $m_2 = 2$ are computed by the numerical carry-over procedure in Tables 14 and 15. The moment coefficients due to $m_3 = 1$ are obtained from symmetry.

In terms of m_j

$$M_1 = +1.105\, m_1 - .387\, m_2 + .105\, m_3$$

$$M_2 = -.331\, m_1 + 1.210\, m_2 - .331\, m_3$$

$$M_3 = +.105\, m_1 - .387\, m_2 + 1.105\, m_3 \,.$$

The final and more suitable form of the influence value equations in terms of

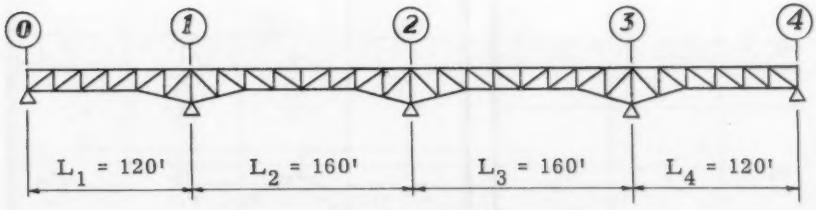
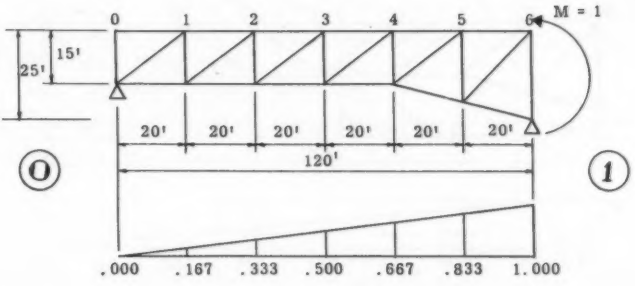


Fig. 13

Four Span Continuous Truss

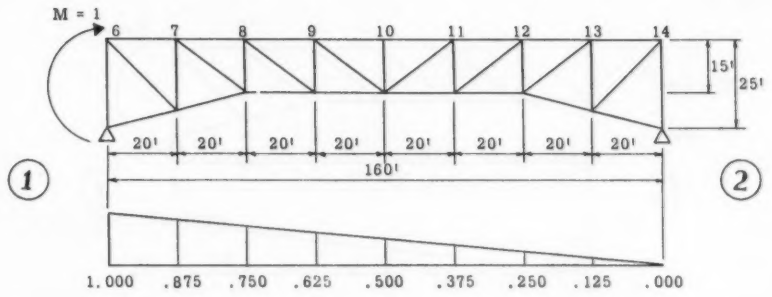
Basic Structure $\overline{01}$ Table 8



	m	λ_m	R'_m	R''_m	β_m	$\beta_m\lambda_m$	P'_m	P''_m
Bottom	1	20.0	15.0		+.0111	+.2220	+.0148	
	2	20.0	15.0		+.0222	+.4440	+.0296	
	3	20.0	15.0		+.0333	+.6660	+.0444	
	4	20.0	15.0		+.0444	+.8880	+.0592	
	5	20.6	19.4		+.0429	+.8837	+.0456	
	6	20.6	24.3		+.0412	+.8487	+.0349	
Top	1	20.0		15.0	.0000	.0000		.0000
	2	20.0		15.0	-.0111	-.2220		+.0148
	3	20.0		15.0	-.0222	-.4440		+.0296
	4	20.0		15.0	-.0333	-.6660		+.0444
	5	20.0		15.0	-.0444	-.8880		+.0592
	6	20.0		20.0	-.0417	-.8340		+.0417
Diagonal	1	25.0	12.0	12.0	-.0139	-.3475	-.0290	+.0290
	2	25.0	12.0	12.0	-.0139	-.3475	-.0290	+.0290
	3	25.0	12.0	12.0	-.0139	-.3475	-.0290	+.0290
	4	25.0	12.0	12.0	-.0139	-.3475	-.0290	+.0290
	5	25.0	12.0	16.0	+.0035	+.0875	+.0073	-.0055
	6	28.3	14.1	17.7	+.0026	+.0736	+.0052	-.0042
Vertical	1	15.0	20.0	20.0	+.0083	+.1250	+.0063	-.0063
	2	15.0	20.0	20.0	+.0083	+.1250	+.0063	-.0063
	3	15.0	20.0	20.0	+.0083	+.1250	+.0063	-.0063
	4	15.0	20.0	20.0	+.0083	+.1250	+.0063	-.0063
	5	20.0	20.0	16.0	-.0021	-.0420	-.0021	+.0026

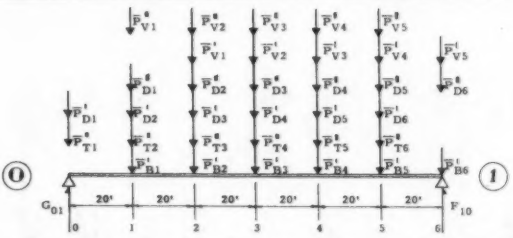
The meaning of symbols R'_m, R''_m, $\beta_m = N_m$, P'_m, P''_m and their analytical expression are shown in Tables 12 and 13 . Value β_m is defined by Eq. (1a) .

Basic Structure T2 Table 9



	m	λ_m	R'_m	R''_m	α_m	$\alpha_m\lambda_m$	P'_m	P''_m
Bottom	7	20.6	24.3		+.0412	+.8487	+.0349	
	8	20.6	19.4		+.0451	+.9291	+.0479	
	9	20.0	15.0		+.0500	+1.0000	+.0667	
	10	20.0	15.0		+.0416	+.8320	+.0556	
	11	20.0	15.0		+.0250	+.5000	+.0333	
	12	20.0	15.0		+.0167	+.3333	+.0222	
	13	20.6	19.4		+.0064	+.1318	+.0068	
	14	20.6	24.3		+.0000			
Top	7	20.0		20.0	-.0438	-.8760		+.0438
	8	20.0		15.0	-.0500	-1.0000		+.0667
	9	20.0		15.0	-.0416	-.8320		+.0556
	10	20.0		15.0	-.0333	-.6667		+.0444
	11	20.0		15.0	-.0333	-.6667		+.0444
	12	20.0		15.0	-.0250	-.5000		+.0333
	13	20.0		15.0	-.0167	-.3333		+.0222
	14	20.0		20.0	-.0062	-.1250		+.0062
Diagonal	7	28.3	14.1	17.7	+.0055	+.1556	+.0110	-.0088
	8	25.0	12.0	16.0	+.0078	+.1950	+.0162	-.0122
	9	25.0	12.0	12.0	-.0104	-.2600	-.0217	+.0217
	10	25.0	12.0	12.0	-.0104	-.2600	-.0217	+.0217
	11	25.0	12.0	12.0	+.0104	+.2600	+.0217	-.0217
	12	25.0	12.0	12.0	+.0104	+.2600	+.0217	-.0217
	13	25.0	12.0	16.0	+.0130	+.3250	+.0271	-.0203
	14	28.3	14.1	17.7	+.0089	+.2519	+.0179	-.0142
Vertical	7	20.0	20.0	16.0	-.0047	-.0940	-.0047	+.0059
	8	15.0	20.0	20.0	+.0063	+.0945	+.0047	-.0047
	9	15.0	20.0	20.0	+.0063	+.0945	+.0047	-.0047
	10	15.0	—	—	—	—	—	—
	11	15.0	20.0	20.0	-.0063	-.0945	-.0047	+.0047
	12	15.0	20.0	20.0	-.0063	-.0945	-.0047	+.0047
	13	20.0	20.0	16.0	-.0078	-.1560	-.0078	+.0098

Conjugate Structure $\overline{0}$ Table 10



$$G_{01} = \sum_{0}^{L} \overline{P}_m \frac{x'_m}{L} = +.1418$$

$$F_{10} = \sum_{0}^{L} \overline{P}_m \frac{x_m}{L} = +.2797$$

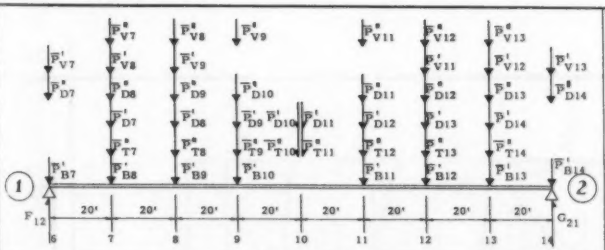
$$\overline{V}_m = G_{01} - \sum_{0}^{x_m} \overline{P}_m$$

$$\overline{M}_m = \overline{M}_{m-1} + \overline{V}_m \ (20)$$

	$\overline{P}'_B$	$\overline{P}'_T$	$\overline{P}'_D$	$\overline{P}^{\#}_D$	$\overline{P}'_V$	$\overline{P}^{\#}_V$	$\Sigma \overline{P}_m$	$\overline{V}_m$	$\frac{\overline{M}_m}{20}$	$M_m = {}^{\#}_{10}$
0		.0000	-.0290				-.0290	+.1418	.0000	.000
1	+.0148	+.0148	-.0290	+.0290		-.0063	+.0233	+.1708	+.1708	+3.416
2	+.0296	+.0296	-.0290	+.0290	+.0063	-.0063	+.0592	+.1475	+.3183	+6.366
3	+.0444	+.0444	-.0290	+.0290	+.0063	-.0063	+.0888	+.0883	+.4066	+8.132
4	+.0592	+.0592	+.0073	+.0290	+.0063	-.0063	+.1547	-.0005	+.4061	+8.122
5	+.0456	+.0417	+.0052	-.0055	+.0063	+.0026	+.0959	-.1552	+.2509	+5.018
6	+.0349			-.0042	-.0021		+.0286	-.2511	+.0000	.000
Σ							+.4215	-.2797		

Functions of the Conjugate Structure: G_{ij}, F_{ji} = Reactions, $\overline{V}_m$ = Shear, $\overline{M}_m$ = Bending Moment

Conjugate Structure $\overline{12}$ Table 11



$$F_{12} = \sum_{0}^{L} \overline{P}_m \frac{x'_m}{L} = +.3904$$

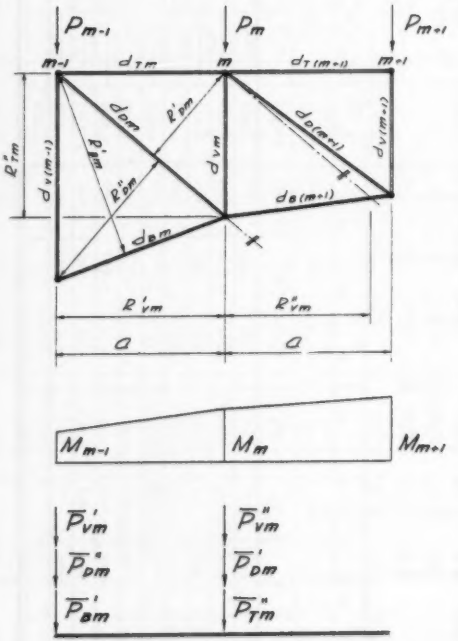
$$G_{21} = \sum_{0}^{L} \overline{P}_m \frac{x_m}{L} = +.2135$$

$$\overline{V}_m = F_{12} - \sum_{0}^{x_m} \overline{P}_m$$

$$\overline{M}_m = \overline{M}_{m-1} + \overline{V}_m \ (20)$$

	$\overline{P}'_B$	$\overline{P}^{\#}_T$	$\overline{P}'_D$	$\overline{P}^{\#}_D$	$\overline{P}'_V$	$\overline{P}^{\#}_V$	$\Sigma \overline{P}_m$	$\overline{V}_m$	$\frac{\overline{M}_m}{20}$	$M_m = {}^{\#}_{12}$
6	+.0349			-.0088	-.0047		+.0214	+.3904	.0000	.000
7	+.0479	+.0438	+.0110	-.0122	+.0047	+.0059	+.1011	+.3690	+.3694	+ 7.388
8	+.0667	+.0667	+.0162	+.0217	+.0047	-.0047	+.1713	+.2679	+.6373	+12.746
9	+.0556	+.0556	-.0217	+.0217		-.0047	+.1065	+.0966	+.7339	+14.678
10		+.0444	-.0217				+.0688	-.0099	+.7240	+14.480
		+.0444	+.0217					-.0987	+.6253	+12.506
11	+.0333	+.0333	+.0217	-.0217		+.0047	+.0713	-.1700	+.4553	+ 9.106
12	+.0222	+.0222	+.0271	-.0217	-.0047	+.0047	+.0498	-.2198	+.2355	+ 4.710
13	+.0068	+.0062	+.0179	-.0203	-.0047	+.0098	+.0157	-.2355	+.0000	.000
14	.0000			-.0142	-.0078		-.0220	-.2135		
Σ							+.6039			

PRATT TRUSS - INTERMEDIATE PANEL TABLE 12



LEVER ARMS

$$R'_{Bm} = \frac{d_{V(m-1)}{}^{a}}{d_{Bm}} \qquad R''_{Tm} = d_{Vm}$$

$$R'_{Dm} = \frac{d_{Vm}{}^{a}}{d_{Dm}} \qquad R''_{Dm} = \frac{d_{V(m-1)}{}^{a}}{d_{Dm}}$$

$$R'_{Vm} = a \qquad R''_{Vm} = \frac{d_{Vm}{}^{a}}{2d_{Vm} - d_{V(m+1)}}$$

AXIAL FORCES

$$N_{Bm} = \frac{M_{m-1}}{R'_{Bm}} \qquad N_{Tm} = -\frac{M_m}{R''_{Tm}}$$

$$N_{Dm} = \frac{M_m}{R'_{Dm}} - \frac{M_{m-1}}{R''_{Dm}} \qquad N_{Vm} = \frac{M_{m-1}}{R'_{Vm}} - \frac{M_m}{R''_{Vm}}$$

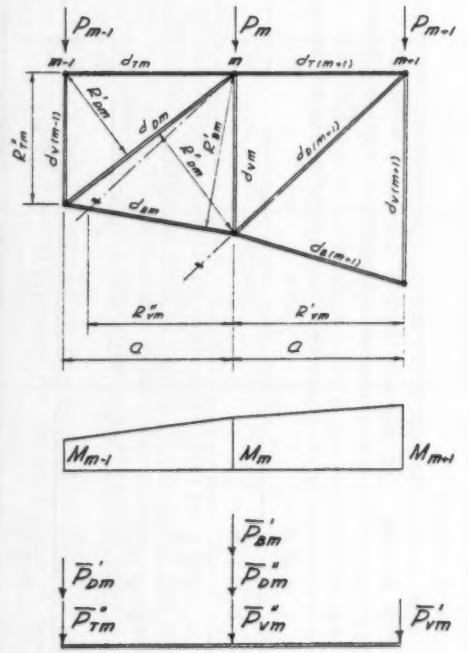
ELASTIC WEIGHTS

$$\overline{P}'_{Bm} = \frac{N_{Bm}\lambda_{Bm}}{R'_{Bm}} \qquad \overline{P}''_{Tm} = -\frac{N_{Tm}\lambda_{Tm}}{R''_{Tm}}$$

$$\overline{P}'_{Dm} = \frac{N_{Dm}\lambda_{Dm}}{R'_{Dm}} \qquad \overline{P}''_{Dm} = -\frac{N_{Dm}\lambda_{Dm}}{R''_{Dm}}$$

$$\overline{P}'_{Vm} = \frac{N_{Vm}\lambda_{Vm}}{R'_{Vm}} \qquad \overline{P}''_{Vm} = -\frac{N_{Vm}\lambda_{Vm}}{R''_{Vm}}$$

PRATT TRUSS - INTERMEDIATE PANEL TABLE 13



LEVER ARMS

$$R'_{Bm} = \frac{d_{Vm}{}^2}{d_{Bm}} \qquad R''_{Tm} = d_{V(m-1)}$$

$$R'_{Dm} = \frac{d_{V(m-1)}{}^2}{d_{Dm}} \qquad R''_{Dm} = \frac{d_{Vm}{}^2}{d_{Dm}}$$

$$R'_{Vm} = a \qquad R''_{Vm} = \frac{d_{Vm}{}^2}{2d_{Vm} - d_{V(m-1)}}$$

AXIAL FORCES

$$N_{Bm} = \frac{M_m}{R'_{Bm}} \qquad N_{Tm} = -\frac{M_{m-1}}{R'_{Tm}}$$

$$N_{Dm} = \frac{M_{m-1}}{R'_{Dm}} - \frac{M_m}{R''_{Dm}} \qquad N_{Vm} = \frac{M_{m+1}}{R'_{Vm}} - \frac{M_m}{R''_{Vm}}$$

ELASTIC WEIGHTS

$$\bar{F}'_{Bm} = \frac{N_{Bm}\lambda_{Bm}}{R'_{Bm}} \qquad \bar{F}''_{Tm} = -\frac{N_{Tm}\lambda_{Tm}}{R''_{Tm}}$$

$$\bar{F}'_{Dm} = \frac{N_{Dm}\lambda_{Dm}}{R'_{Dm}} \qquad \bar{F}''_{Dm} = -\frac{N_{Dm}\lambda_{Dm}}{R''_{Dm}}$$

$$\bar{F}'_{Vm} = \frac{N_{Vm}\lambda_{Vm}}{R'_{Vm}} \qquad \bar{F}''_{Vm} = -\frac{N_{Vm}\lambda_{Vm}}{R''_{Vm}}$$

Moments Due to $m_1 = +1$ — Table 14

Supports	1	2	3
Carry-Over Factors	$\overrightarrow{-.273}$	$\overleftarrow{-.319}$ $\overrightarrow{-.319}$	$\overleftarrow{-.273}$
Starting Moment	+1.000		
1. C.O.M.		-.273	
2. C.O.M.	+.087		+.087
3. C.O.M.		-.024 -.024	
4. C.O.M.	+.015		+.015
5. C.O.M.		-.004 -.004	
6. C.O.M.	+.003		+.003
7. C.O.M.		-.001 -.001	
Final Moments	+1.105	-.331	+.105

Moments Due to $m_2 = +1$ — Table 15

Supports	1	2	3
Carry-Over Factors	$\overrightarrow{-.273}$	$\overleftarrow{-.319}$ $\overrightarrow{-.319}$	$\overleftarrow{-.273}$
Starting Moment		+1.000	
1. C.O.M.	-.319		-.319
2. C.O.M.		+.087 +.087	
3. C.O.M.	-.056		-.056
4. C.O.M.		+.015 +.015	
5. C.O.M.	-.010		-.010
6. C.O.M.		+.003 +.003	
7. C.O.M.	-.002		-.002
Final Moments	-.387	+1.210	-.387

$$m_j = -\frac{\Sigma \tau_j}{\Sigma F_j}$$

is:

$$\left.\begin{aligned} M_1 &= -1.649\,\Sigma\tau_1 + .494\,\Sigma\tau_2 - .157\,\Sigma\tau_3 \\ M_2 &= +.494\,\Sigma\tau_1 - 1.551\,\Sigma\tau_2 + .494\,\Sigma\tau_3 \\ M_3 &= -.157\,\Sigma\tau_1 + .494\,\Sigma\tau_2 - 1.649\,\Sigma\tau_3 . \end{aligned}\right\} \quad (16)$$

The same but somewhat more precise results may be obtained by means of algebraic coefficients (Table 7).

(d) End Slope Influence Values:

The end slope influence line of a simple truss is represented by the elastic curve of an equivalent beam, due to a unit moment applied at the respective end (Maxwell's Theorem), or the bending moment curve of a simple beam conjugated to the first one (Mohr's Theorem). Thus the M_m's in Table 10 and 11 are the relative values of the respective slopes.

(e) Moment Influence Values:

The influence values for M_1, M_2, and M_3 may now easily be obtained by substituting the values of τ's from Tables 10 and 11 into the final form of moment equations 16 as shown in Tables 16 and 17.

ACKNOWLEDGMENT

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Appendix

For the derivation of the general three moment Eq. (4a) for a truss of variable depth, reference is given to work of Bažant.(6) Recently, this equation was restated by Wei.(7) The advantages of the three moment equation solution over some other methods of truss analysis were clearly demonstrated by Mullins.(8)

The constants F's, G's and τ's may easily be computed by means of elastic weights. The derivation of general formulas for elastic weights is shown in works of Mohr,(9) Bažant,(10) Timoshenko,(11) Stüssi(12) and Stabilini.(13) For the convenience of the reader, tables of elastic weights for the Pratt truss are included in this paper. A more elaborate set of tables of elastic weights is being prepared by Sturm and the writer.(14)

Example — Load in First or Fourth Span - Influence Values M_1, M_2, M_3 — Table 16

m	τ_{10}	M_1	M_2	M_3	
		$-1.649\ \tau_{10}$	$+.494\ \tau_{10}$	$-.157\ \tau_{10}$	
0	.000	.000	.000	.000	28
1	+3.416	- 5.632	+1.688	- .536	27
2	+6.366	-10.498	+3.145	- .999	26
3	+8.132	-13.410	+4.017	-1.277	25
4	+8.122	-13.393	+4.012	-1.275	24
5	+5.018	- 8.274	+2.479	- .788	23
6	.000	.000	.000	.000	22
	τ_{34}	M_3	M_2	M_1	m

Example — Load in Second or Third Span - Influence Values M_1, M_2, M_3 — Table 17

m	τ_{12}	τ_{21}	M_1			M_2			M_3			
			$-1.649\ \tau_{12}$	$+.494\ \tau_{21}$	Σ	$+.494\ \tau_{12}$	$-1.551\ \tau_{21}$	Σ	$-.157\ \tau_{12}$	$+.494\ \tau_{21}$	Σ	
6	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	22
7	+ 7.388	+ 4.710	-12.183	+2.327	- 9.856	+3.650	- 7.305	- 3.655	-1.160	+2.327	+1.167	21
8	+12.746	+ 9.106	-21.018	+4.498	-16.520	+6.297	-14.123	- 7.826	-2.001	+4.498	+2.497	20
9	+14.678	+12.506	-24.204	+6.178	-18.026	+7.251	-19.397	-12.146	-2.304	+6.178	+3.874	19
10	+14.480	+14.480	-23.878	+7.153	-16.725	+7.153	-22.458	-15.305	-2.273	+7.153	+4.880	18
11	+12.506	+14.678	-20.622	+7.251	-13.371	+6.178	-22.766	-16.588	-1.963	+7.251	+5.288	17
12	+ 9.106	+12.746	-15.015	+6.297	- 8.718	+4.498	-19.769	-15.271	-1.430	+6.297	+4.867	16
13	+ 4.710	+ 7.388	- 7.766	+3.650	- 4.116	+2.327	-11.458	- 9.131	- .739	+3.650	+2.911	15
14	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	14
	τ_{32}	τ_{23}	M_3			M_2			M_1			m

The calculation of elastic weights for the top and bottom members is relatively simple. The calculation of elastic weights for web members is, however, more laborious. It must be noted that for the calculation of redundants only relative values of F's, G's and τ's are significant. By neglecting the influence of the web member elastic weights, large errors are introduced in the actual slopes (20 to 50 percent), whereas the effect of the web member elastic weights on the relative slopes is small (3 to 10 per cent). The cross-sectional areas of truss members "which cannot be found without knowing them first" are the common dilemma of all indeterminate truss analysis.[15] Haertlein's investigations[16,17] on several well known existing structures show that with the simplification

$$\lambda_m = \frac{d_m}{A_m E} = 1$$

$$\lambda_n = \frac{d_n}{A_n E} = 1$$

excellent results may be obtained in continuous trusses of conventional type. Thus it becomes practical (particularly in trusses with panels of equal length) to assume:

$$F_{ji} = \sum_{i}^{j} \beta_m^2 \qquad F_{jk} = \sum_{j}^{k} \alpha_n^2 \tag{5'}$$

$$G_{ij} = \sum_{i}^{j} \alpha_m \beta_m \qquad G_{jk} = \sum_{j}^{k} \alpha_n \beta_n \tag{6'}$$

$$\tau_{ji} = \sum_{i}^{j} BN_m \beta_m \qquad \tau_{jk} = \sum_{j}^{k} BN_n \alpha_n \; . \tag{7'}$$

In light of this discussion, the error in splitting the vertical bars and the omission of the elastic weights of these members in Tables 8, 9, 10, 11 becomes insignificant.

Tables of

ELASTIC WEIGHTS

For

Pratt Trusses

Nomenclature - Tables 12 & 13

Major Symbols

a, b, c, d = Dimensions
f = Diagonal Lever Arm for a truss of Constant Depth
h = Constant Depth of Truss
M = Bending Moment
N = Axial Force
P = Load
$\overline{P}$ = Elastic Weight
R = Lever Arm
λ = Axial flexibility
ω = Slope of a Bar

Subscripts

m = Panel
B = Bottom
D = Diagonal
T = Top
V = Vertical

REFERENCES

1. "Analysis of Continuous Beams by Carry-Over Moments" by Jan J. Tuma, Proceedings, A.S.C.E., Vol. 84, September, 1958, No. 1762.
2. "Analysis of Continuous Frames by Distributing Fixed End Moments," by Hardy Cross, Transactions, A.S.C.E., Vol. 97, 1932, pp. 1-10.
3. "Analysis of Continuous Frames by Balancing Angle Changes," by L. E. Grinter, Transactions, A.S.C.E., Vol. 102, 1937, p 1020.
4. "Determining Influence Lines by Balancing Angle Changes," by L. E. Grinter, Engineering News Record, Vol. 121, 1938, p 443.
5. "Influence Lines by Corrections to an Assumed Shape," by J. P. Michalos and E. N. Wilson, Proceedings, A.S.C.E., Vol. 78, April, 1952, No. 124.
6. "Stavebná Mechanika," by Z. Bažant, Vol. III, 3rd Ed., Praha, 1955, pp. 76-91.
7. "Analysis of Multiple-Span Continuous Trusses," by B. C. F. Wei, Proceedings, A.S.C.E., Vol. 83, March, 1957, No. 1187.
8. "Ibid, Discussion," H. H. Mullins, Proceedings, A.S.C.E., Vol. 83, September, 1957, No. 1382-45.

9. "Beitrag Zur Theorie Des Fachwerkes" by O. Mohr, Zeitschrift der Architekten-Ingenieur-Verein, Hannover, 1875, p 17.

10. "Stavebná Mechanika," by Z. Bažant, Vol. II, 4th Ed., Praha, 1954, pp. 356-365.

11. "Theory of Structures," by S. Timoshenko and D. H. Young, New York, 1945, pp. 277-285.

12. "Vorlesungen Űber Baustatik" by F. Stűssi, Vol. II, Basel, 1954, pp. 46-48.

13. "Tecnica Delle Construzioni," by L. Stabilini, Vol. I, Milano, 1956, pp. 334-343.

14. "Tables of Elastic Weights," by Jan. J. Tuma and E. R. Sturm, Oklahoma Engineering Experiment Station Publication, Oklahoma State University, Stillwater, (in preparation).

15. "Moment Distribution and the Analysis of Truss of Varying Depth," by E. R. Jacobsen, Engineering Journal, Engineering Institute of Canada, December, 1940, pp. 503-506.

16. "The Design of Statically Indeterminate Trusses," by Albert Haertlein, Journal of the Boston Society of Civil Engineers, April, 1936.

17. "Analysis of Statically Indeterminate Structures," by J. I. Parcel and R. B. Moorman, New York, 1955, pp. 333-335.

18. "Influence Lines for Continuous Beams," by K. S. Havner and J. J. Tuma, Oklahoma Engineering Experiment Station Publication, Oklahoma State University, Stillwater, No. 106, 1959.

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VIERENDEEL BENTS WITH NONPRISMATIC MEMBERS

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SYNOPSIS

Inclined-chord Vierendeel bents with members of variable section are analyzed by means of a modified moment-distribution procedure in which the equilibrium between the internal forces in the chord members and the external forces is maintained throughout the entire distribution operation. The solution is obtained directly with the aid of relatively simple expressions for the fixed-end moments, the stiffnesses and the carry-over factors.

Notation

a	ratio of length of the far cross member to length of near cross member
C	carry over factor from near end to far end of member, with far end fixed, in ordinary moment distribution
C'	carry over factor from far end to near end of member, with near end fixed, in ordinary moment distribution
C_1	carry over factor for chord member
d	deflections normal to axis of member
d_1, d_2	deflections
E	modulus of elasticity
f_a, f_b, f_c	forces in force systems a, b, and c respectively
F	arbitrary force

Note: Discussion open until May 1, 1960. To extend the closing date one month, a written request must be filed with the Executive Secretary, ASCE. Paper 2321 is part of the copyrighted Journal of the Structural Division, Proceedings of the American Society of Civil Engineers, Vol. 85, No. ST 10, December, 1959.

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I	moment of inertia
I_o	moment of inertia at reference section
k	stiffness factor of near end of member, with far end fixed, in ordinary moment distribution
k'	same item for far end of member
K	EI/L
K_o	EI_o/L
L	length of member
M	moment acting on near end of chord member, positive if clockwise
M'	same item for far end of member
M_E	one half of the moment about the near end of a chord member caused by all external forces acting on the free body on the near side of the structure cut by a section passing through the end under consideration and parallel to the cross members, positive if acting clockwise; or one half of the moment caused by all the forces acting on the free body on the far side of the structure cut by the same section, positive if acting counter-clockwise
M'_E	same item for far end of member
M_F	fixed end moment acting on near end of member
M'_F	same item for far end of member
P	reactive force acting on chord member in direction normal to cross members
Q_1, Q_2 ---	external forces
q_1, q_2 ---	perpendicular distances of Q_1, Q_2 --- respectively from a joint
r	distribution factor
S_1	stiffness of chord member
S_2	stiffness of cross member
θ	rotation of near end of member, positive if clockwise
θ'	rotation of far end of member, positive if clockwise

INTRODUCTION

In the analysis of Vierendeel bents by ordinary moment distribution method,(1) the rotation of each joint in the distribution procedure introduces unbalanced internal forces. To maintain equilibrium and to prevent joint translation, fictitious external forces must be introduced. In order to remove these forces, it is necessary to deal with several auxiliary force systems and to solve simultaneous equations. The final solution is obtained by superimposing the several force systems.(2)*

*See bibliographical listing, p. 90.

In the modified moment-distribution method discussed in the following, only the actual force system needs be considered. The analysis consists of first determining the fixed end moments acting on the chord members in each panel such that these moments and the corresponding shearing and normal forces are in equilibrium with the external forces, without regards to the equilibrium of the joints. Subsequently the joints are balanced in pairs by means of a moment distribution procedure in which the bending moments and the corresponding shearing and normal forces introduced in the chord members are such that they are in equilibrium with zero external forces. Thus the equilibrium between the internal forces in the chord members and the external forces is maintained throughout the entire balancing operation. Therefore once the equilibrium of the joints are attained, the desired distribution of bending moments in the structure is obtained directly.

This concept was first proposed by Maugh[3] who discussed Vierendeel trusses with prismatic members. Historical reviews of the subject were given in the discussion of a paper by Grinter and Tsao[4] and also in the introduction of a paper by Lightfoot.[5] A dissertation by Tsao[6] prompted the present study; the expressions derived in the following degenerate into the expressions given in his work for Vierendeel bents with prismatic members. Lightfoot presented essentially the same method, but again his paper dealt only with prismatic members. The present paper may be regarded as a simplification and extension of the above mentioned work.

The proposed method is applicable to rigid frames composed of successive rectangular or trapezoidal panels with all external forces applied at the joints (Fig. 1). The only restrictions are that the corresponding pairs of chord members must have equal flexural properties and the cross members must be symmetrical and parallel to one another. The chord members may be of unequal lengths. If a prismatic chord member is longer than its counterpart on the opposite side, it must have a larger moment of inertia so that the value of $K = EI/L$ will be the same. If they are haunched, the length of the haunch must be in the same proportion to the total length of the corresponding member and the variation of the moment of inertia must follow the same pattern. The ratio of the minimum and maximum moment of inertia as well as EI_o/L must be the same for both members. In the latter, E is the modulus of elasticity, I_o the moment of inertia at the reference section, and L denotes the length of the member.

Geometry

In a typical panel such as ABCD shown in Fig. 2, if C and D are rotated simultaneously an equal amount and the far ends of the chord members,* A and B, are allowed to displace as if the cross member AB were infinitely rigid but otherwise free to displace as a rigid body, point A moves to A', point B to B' and AB rotates through an angle θ'. It can be seen that the center of rotation of this mode of panel deformation is point O, shown in Fig. 3.

It follows from similar triangles that

*In this discussion, C and D are considered the near ends of the chord members BC and AD respectively, and A and B the far ends respectively.

$$\frac{\overline{AB}}{\overline{CD}} = \frac{\overline{AO}}{\overline{DO}} = \frac{\overline{AA'}}{\overline{DD'}}$$

Introducing $\overline{AA'} = d$, $\overline{AD} = L$ and $\overline{DD'} = d + \theta' L$, the above relationship yields

$$\frac{\overline{AB}}{\overline{CD}} = \frac{d}{d + \theta' L}$$

from which

$$\frac{d}{L} = \frac{(\overline{AB}/\overline{CD})}{1 - (\overline{AB}/\overline{CD})}\,\theta'$$

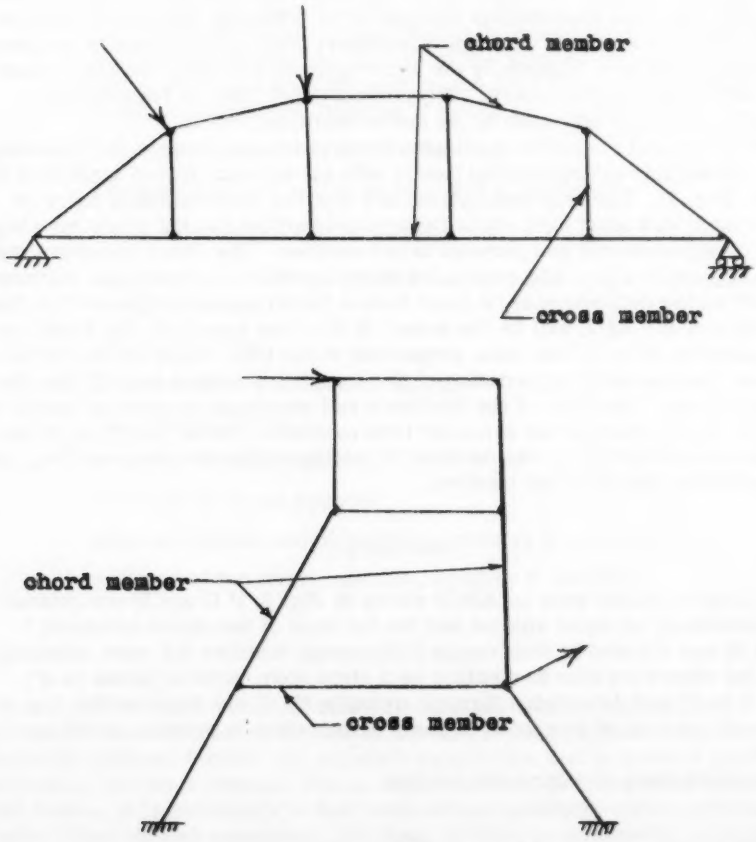


Fig. 1. Inclined chord Vierendeel bents.

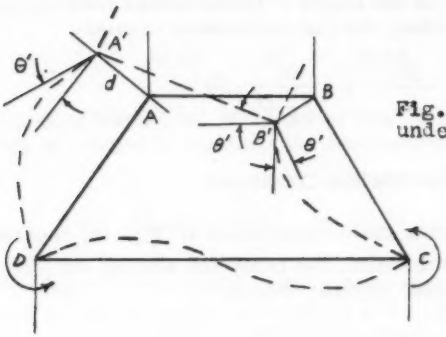


Fig.2 Deformation of panel under consideration

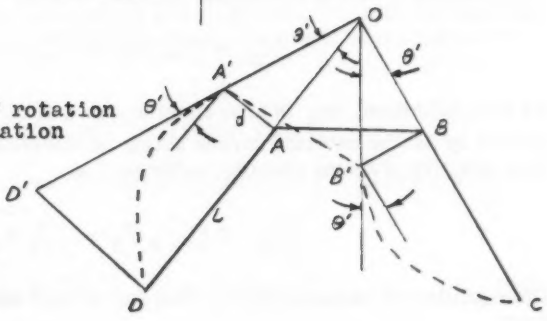
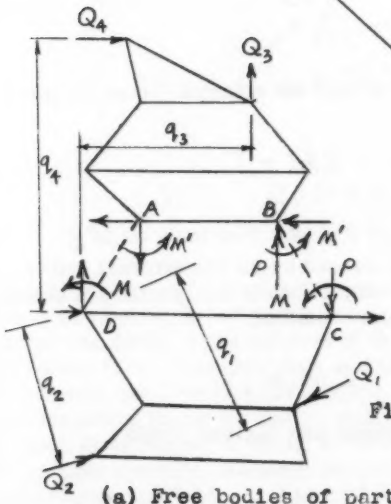


Fig.3 Center of rotation of panel deformation



(a) Free bodies of parts of structure separated by panel under consideration

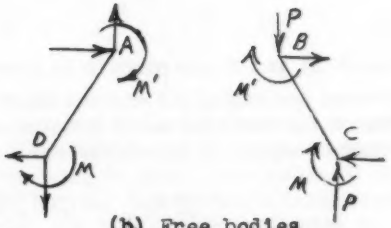


Fig.4 (b) Free bodies of chord members

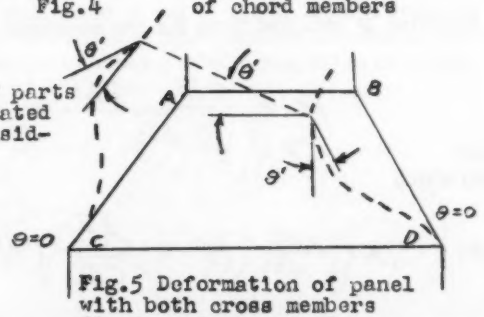


Fig.5 Deformation of panel with both cross members fixed against rotation

Setting $a = \overline{AB}/\overline{CD}$, the ratio of the length of the far cross member to the length of the near cross member, the last expression becomes

$$\frac{d}{L} = \frac{a}{1-a}\theta' \qquad (1)$$

Equilibrium Conditions

Let M_E represent one half of the moment about D (Fig. 4a) caused by all the external forces acting on the structure below the section cut just above CD, positive if acting clockwise. In other words,

$$M_E = (Q_1 q_1 - Q_2 q_2)/2$$

By this definition, M_E is also equal to one half of the moment about D caused by all the external forces acting on the structure above the same section, positive if acting counterclockwise; i.e.,

$$M_E = (Q_3 q_3 - Q_4 q_4)/2$$

The equilibrium between the internal forces and the external forces requires that*

$$2M_E + P\,\overline{CD} - 2M = 0$$

in which M and P are positive as shown in Fig. 4b. Similarly let M'_E represent one half of the moment about A caused by all the external forces acting on the structure above the section cut just below AB, positive if acting clockwise. Again, to satisfy the equilibrium condition,

$$2M'_E - P\,\overline{AB} - 2M' = 0$$

Equating P obtained from the two equations just derived yields

$$P = \frac{2M'_E - 2M'}{\overline{AB}} = \frac{2M_E + 2M}{\overline{CD}}$$

from which

$$M'_E - M' = \frac{\overline{AB}}{\overline{CD}}(-M_E + M)$$

*The corresponding chord members have the same flexural properties. It can be shown that, in this case, the bending moments acting on A and B (or C and D) are equal.

Substituting $a = \overline{AB}/\overline{CD}$ leads to

$$aM + M' = aM_E + M'_E \tag{2a}$$

Eq. (2a) is applicable to final moments as well as to fixed end moments, hence it may be written in terms of the fixed end moments M_F and M'_F in the form

$$aM_F + M'_F = aM_E + M'_E \tag{2b}$$

The terms "fixed end moments" used in this paper refer to the bending moments acting on the chord members which are in equilibrium with the external loads if the cross members were infinitely rigid. This will be discussed in the following.

Fixed End Moments for Typical Panel

It can be shown* that the slope-deflection equations for a member with variable moment of inertia are

$$M = k\left[\theta + C\theta' + (1+C)\frac{d}{L}\right]K_o \tag{3a}$$

$$M' = Ck\left[\theta + \frac{1}{C'}\theta' + \left(1+\frac{1}{C'}\right)\frac{d}{L}\right]K_o \tag{3b}$$

in which M is the moment acting on the near end of the chord member, positive if clockwise, k the stiffness factor of the member at the near end with the far end fixed, θ the rotation of the near end, positive if clockwise, C the carryover factor from the near end to the far end, d the relative translation of the ends, positive as shown in Fig. 2, $K_o = E_oI/L$ and prime denotes the corresponding items for the far end. Both k and D are as used in ordinary moment distribution, C being always positive since bending moment is taken as positive if acting clockwise on the member. Substituting (1) in the above expressions yields

$$M = k\left\{\theta + \theta'\left[C + (1+C)\frac{a}{1-a}\right]\right\}K_o \tag{4a}$$

$$M' = Ck\left\{\theta + \theta'\left[\frac{1}{C'} + \left(1+\frac{1}{C'}\right)\frac{a}{1-a}\right]\right\}K_o \tag{4b}$$

*See (2), p. 132, where different notation is used. Also see (7), (8), (9), and (10) for tables of stiffness factors, carry-over factors, and fixed end moments.

To obtain expressions for the fixed end moments, consider CD (Fig. 5) to be stationary while AB is allowed to displace until the internal forces in the chord members are in equilibrium with the external loads. Thus setting $\theta = 0$ in (4a, b) leads to

$$M_F = k\theta'\left[C + (1+C)\frac{a}{1-a}\right]K_o \tag{5a}$$

$$M'_F = Ck\theta'\left[\frac{1}{C'} + \left(1+\frac{1}{C'}\right)\frac{a}{1-a}\right]K_o \tag{5b}$$

Substituting (5a, b) in (2b) leads to

$$aM_E + M'_E = k\theta'\left[Ca + (1+C)\frac{a^2}{1-a}\right]K_o + k\theta'\left[\frac{C}{C'} + \left(C + \frac{C}{C'}\right)\frac{a}{1-a}\right]K_o$$

from which

$$K_o k\theta' = \frac{aM_E + M'_E}{Ca + \frac{a^2}{1-a} + C\frac{a^2}{1-a} + \frac{C}{C'} + C\frac{a}{1-a} + \frac{C}{C'}\frac{a}{1-a}}$$

Substituting the last expression in (5a) yields the fixed end moment

$$M_F = \frac{(aM_E + M'_E)(a+C)}{a^2 + 2Ca + C/C'} \tag{6a}$$

For a prismatic chord member, $C = C' = 1/2$ and (6a) becomes

$$M_F = \frac{(aM_E + M'_E)(a+1/2)}{a^2 + a + 1} \tag{6b}$$

If the loads are not parallel to the cross members, M_E computed for the joint at one end of the cross member will not be the same as that computed for the joint at the other end. It can be shown, however, that $aM_E + M'_E$ will be the same for both cases. When computing this item, it is best to proceed along one or the other chord from joint to joint.

No fixed end moments act on the cross members.

Fixed End Moments for Special Panels

For joints at the cross member in a triangular panel, $a = 0$ since the length of the far cross member is zero. For this case, (6a) becomes

$$M_F = M'_E C' \tag{7}$$

If no external moment is applied at the vertex M'_E will be zero, hence $M_F = 0$.

For the joint at the vertex of the triangle, setting $a \to \infty$ in (6a) yields

$$M_F = M_E \tag{8}$$

and, if no external moment is applied at this joint, M_E will be zero, hence, $M_F = 0$.

For a panel with hinged base, the fixed end moment at the hinged base $M_F = 0$ and, for the other end of the chord member, setting $M'_F = 0$ in (2b) leads to

$$M_F = \frac{M'_E}{a} + M_E \tag{9}$$

For a panel with fixed base, the fixed end moments are computed as for the typical panel.

The fixed end moments, as well as the stiffness and the carry-over factor treated in the following, are summarized in Table 1.

Stiffness and Carry Over Factor for Typical Panel

The flexural stiffness of the near end of a member may be defined as the bending moment required to produce unit rotation at the same end. It should be remembered that the external loads are kept in equilibrium by the fixed end moments on the chord members. In order not to disturb this equilibrium, the internal forces introduced by the subsequent balancing of the joints should be in equilibrium with zero external load. Perhaps an appropriate name for this distribution procedure is the "zero load moment distribution." To obtain an expression for the stiffness of a chord member which satisfies the above requirement, M_E and M'_E are set equal to zero in (2a) which takes the form

$$aM + M' = 0 \tag{10}$$

Substituting (4a, b) in (10) and setting $\theta = 1$ lead to

$$k\left\{a+\theta'\left[Ca+(1+C)\frac{a^2}{1-a}\right]\right\}K_0 + k\left\{C+\theta'\left[\frac{C}{C'}+\left(\frac{C}{C'}+C\right)\frac{a}{1-a}\right]\right\}K_0 = 0$$

from which

$$\theta' = -\frac{(C+a)(1-a)}{a^2+2Ca+C/C'}$$

Substituting the last expression in (4a) and again setting $\theta = 1$ yield the stiffness of the chord member.

$$S_1 = Ck\,\frac{1/C' - C}{a^2+2Ca+C/C'}\,K_0 \tag{11a}$$

	Panel type	End	M_F	S_1 or S_2	r	C_1
Chord members	Typical	At either end	$\frac{(aM_E+M'_E)(a+C)}{a^2+2Ca+C/C'}$	$Ck\frac{1/C'-C}{a^2+2Ca+C/C'}K_o$	compute	$-a$
	Fixed base	At fixed base	same as typical panel	not needed	0	not needed
		At other end	same as typical panel			
	Hinged base	At hinged base	0	not needed		
		At other end	$(M'_E/a)+M_E$	0	0	0
	Triangular	At vertex	M_E	not needed		
		At cross member	M'_EC'	$k(1-CC')K_o$	compute	0
Cross members			0	$k(1+C)K_o$	compute	0

For prismatic members, C=C'=1/2 and k=4
For parallel-chord panels, a=1

Table 1 Fixed end moments, stiffnesses, distribution factors and carry-over factors

For prismatic chord members, k = 4 and C = C' = 1/2 for which (11a) becomes*

$$S_1 = \frac{3}{a^2+a+1}K \tag{11b}$$

The carry over factor, by (10), is

$$C_1 = \frac{M'}{M} = -a \tag{12}$$

Since the cross members undergo antisymmetrical bending, their stiffnesses are

*The subscript in K_o is dropped for prismatic members.

$$S_2 = k(1+C)K_o \tag{13a}$$

For prismatic cross members,

$$S_2 = 6K \tag{13b}$$

The carry over factor for cross members is of course zero, since both ends of the member are rotated through the same angle simultaneously in anti-symmetrical bending.

Stiffness and Carry Over Factor for Special Panels

For joints at the cross member in a triangular panel, setting a = 0 in (11a) yields

$$S_1 = k(1-CC')K_o \tag{14a}$$

For prismatic chord members, (14a) becomes

$$S_1 = 3K \tag{14b}$$

This is the same as the stiffness of a member hinged at the far end in ordinary moment distribution. It can be equally well established from the following argument. The vertex of a triangular frame is restrained against translation if the base were fixed in position. The moments acting on the two chord members at the vertex of the triangle would have to be equal and opposite in sign, in this case, a condition which is possible only if the moments are zero. The carry over factor to the vertex is equal to zero since a = 0 in (12).

For a panel with fixed base the distribution factor at the fixed base is zero. The stiffness of the other end of the chord member is computed as for a typical panel. The carry over factor to the fixed base is "-a" as given by (12).

For a panel with hinged base the carry over factor to the hinged base is zero. The stiffness of the other end of the chord member is also zero, a fact which can be verified by setting C = 0 in (11a).

Example 1

The Vierendeel truss shown in Fig. 6 will be used to illustrate the application of the proposed method of analysis. This example is taken from reference (5) where a history of the problem can be found. The K values for all the members are equal.

Table 2 shows the computation required to obtain the fixed end moments. Row 8 of Table 2, by comparison with Row 4, gives a simple check on the fixed end moments in accordance with Eq. (2b). Row 9 is calculated only after the rest of the computation is completed and provides simple check on the final moments in accordance with Eq. (2a).

Table 3 shows the determination of the distribution factors which are obtained as in ordinary moment distribution, and Table 4 shows the distribution operation. The cross members could have been omitted in the latter and the final end moments acting on them obtained from the equilibrium of the joints.

Table 2 Computation of fixed-end moments for Example 1
All quantities in feet and pounds

	Member	AB	BA	BC	CB	CD	DC	DE	ED	EF	FE
1	a	2	0.5	1.25	.8	1	1	0.8	1.25	0.5	2
2	M_E	0	+3000	-3000	+6000	-6000	+4000	-4000	+2000	-2000	0
3	M'_E	+3000	0	+6000	-3000	+4000	-6000	+2000	-4000	0	-2000
4	$aM_E+M'_E$	+3000	+1500	+2250	+1800	-2000	-2000	-1200	-1500	-1000	-2000
5	a+.5	2.5	1	1.75	1.3	1.5	1.5	1.3	1.75	1	2.5
6	a^2+a+1	7	1.75	3.81	2.44	3	3	2.44	3.81	1.75	7
7	M_F	+1071	+857	+1032	+959	-1000	-1000	-639	-688	-572	-714
8	$aM_F+M'_F$	+2999	+1500	+2249	+1799	-2000	-2000	-1200	-1499	-1000	-2000
9	$aM+M'$	+2999	+1499	+2250	+1800	-2000	-2000	-1200	-1500	-999	-1998

Example 2

For the second example, consider again the Vierendeel truss shown in Fig. 6 but, in this instance, assume that all the members are infinitely rigid for a distance equal to one-tenth of the length of the members from the joints and that their K_o values are equal. For this case, the tables in reference (8) give C = C' = 0.648 and k = k' = 7.11 and the numerical work required to obtain a solution is given in Tables 5, 6 and 7. The difference between the final moments obtained in this example and the results of Example 1 is given at the bottom of Table 7. It can be seen that the difference is considerable* and that it cannot be ignored in design.

Example 3

The distribution of bending moments in a Vierendeel bent loaded at the panel points is such that a more economical design can be achieved by making the members more rigid at the ends than at the center. It is of interest to consider again, as the third example, the Vierendeel truss shown in Fig. 6, but, in the present case, assume the moment of inertia of the two end thirds

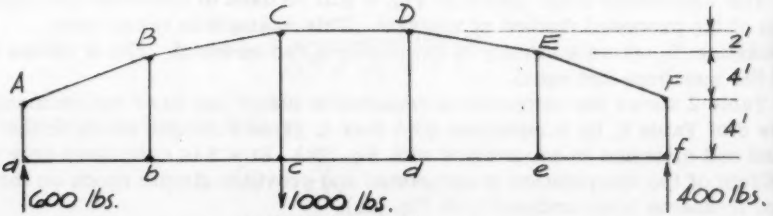


Fig. 6. Vierendeel truss used in Examples 1, 2 and 3.

*See also Example 4.

Joint	A		B			C			D			E			F	
Member	Aa	AB	BA	Bb	BC	CB	Cc	CD	DC	Dd	DE	ED	Ee	EF	FE	Ff
C_1		-2 →	←.5		-1.25 →	←.8		-1 →	←1		-.8 →	←1.25		-.5 →	←2	
r	.933	.067	.202	.705	.093	.150	.728	.122	.122	.728	.150	.093	.705	.202	.067	.933
M_F		+1071	+857		+1032	+959		-1000	+1000		-639	-688		-572	-71.4	
		+190	-380	-1334	-175	+219		-199	+199	+1195	+245	-196				
	-1177	-84	+168								-169	+135	+1027	+294	-147	
		+17	-34	-118	-16	+20		-21	+21	+123	+25	-20		-114	+57	+804
	-16	-1	+2								-15	+12	+95	+27	-13	
								-2	+2	+11	+2	-2		-2	+1	+12
					-3	+4	+17	+3	-3		0	0	+3	+1	0	
		0	0	+1	0				0	+3	0					
M	-1193	+1193	+613	-1451	+838	+1202	+17	-1219	-781	+1332	-551	-759	+1125	-366	-816	+816

Table 4 Distribution operation for Example 1
All quantities in feet and pounds

Joint	Member	S_1 or S_2			r
A	Aa			6.00K	.933
	AB	(3/7)K	=	0.43K	.067
				6.43K	1.000
B	BA	(3/1.75)K	=	1.72K	.202
	Bb			6.00K	.705
	BC	(3/3.81)K	=	0.79K	.093
				8.51K	1.000
C	CB	(3/2.44)K	=	1.23K	.150
	Cc			6.00K	.728
	CD	(3/3)K	=	1.00K	.122
				8.23K	1.000

Values for joints D, E and F are obtained by symmetry

Table 3 Distribution factors for Example 1

	Member	AB	BA	BC	CB	CD	DC	DE	ED	EF	FE
1	a	2	.5	1.25	.8	1	1	.8	1.25	.5	2
2	M_E	0	+3000	-3000	+6000	-6000	+4000	-4000	+2000	-2000	0
3	M'_E	+3000	0	+6000	-3000	+4000	-6000	+2000	-4000	0	-2000
4	$aM_E+M'_E$	+3000	+1500	+2250	+1800	-2000	-2000	-1200	-1500	-1000	-2000
5	a+C	2.648	1.148	1.898	1.448	1.648	1.648	1.448	1.898	1.148	2.648
6	$a^2+2Ca+C/C'$	7.592	1.898	4.183	2.677	3.296	3.296	2.677	4.183	1.898	7.592
7	M_F	+1047	+907	+1020	+975	-1000	-1000	-650	-680	-604	-698
8	$aM_F+M'_F$	+3001	+1500	+2250	+1800	-2000	-2000	-1200	-1500	-1000	-2000
9	$aM+M'$	+3001	+1500	+2251	+1801	-1999	-1999	-1199	-1499	-1000	-1999

Table 5 Computation of fixed end moments for Example 2
All quantities in feet and pounds

Joint	Member	S_1 or S_2	$S_1/7.11$ or $S_2/7.11$	r
A	Aa	$7.11(1+.648)K_o$ =	$1.648K_o$	.956
	AB	$\{[(.648)(7.11)(1/.648-.648)]/7.592\}K_o$ =	$.076K_o$	.044
			$1.724K_o$	1.000
B	BA	$\{[(.648)(7.11)(1/.648-.648)]/1.898\}K_o$ =	$.305K_o$	.147
	Bb	same as Aa	$1.648K_o$	.786
	BC	$\{[(.648)(7.11)(1/.648-.648)]/4.183\}K_o$ =	$.139K_o$	.067
			$2.072K_o$	1.000
C	CB	$\{[(.648)(7.11)(1/.648-.648)]/2.677\}K_o$ =	$.216K_o$	.106
	Cc	same as Aa	$1.648K_o$	.808
	CD	$\{[(.648)(7.11)(1/.648-.648)]/3.296\}K_o$ =	$.176K_o$	.086
			$2.040K_o$	1.000

Values for joints D, E and F are obtained by symmetry

Table 6 Distribution factors for Example 2

of each member to be three times that of the middle third and that the K_o values for all members are again equal. Such a case could conceivably occur in a welded steel member with cover plates. For this case, C = C' = .647 and k = k' = 10.18. Since the carry-over factor is approximately the same as that in the case of Example 2, it is obvious from Eq. (6a) that the corresponding fixed-end moments will be identical for all practical purposes. Furthermore, since the values of C, C', k and K_o are the same for all members as in Example 2, it follows from Eq. (11a) and (13a) that the distribution factors in this case will be the same as those of Example 2. Hence the final moments will also be identical.

Example 4

To illustrate the treatment of triangular panel and panel with hinged base, consider the frame shown in Fig. 7a. All the members are one foot in width and the dimensions and flexural properties of pertinent members are indicated in Fig. 7b. The calculation of the fixed-end moments and the distribution factors are shown in Tables 8 and 9 respectively, and the distribution operation in Table 10. It should be noted that M_E at BC is not equal to $-M_E$ at BA because of the presence of the two-hundred pound load applied normal to the cross member and M_E and M'_E are computed at joint B instead of b. The final moments are shown at the bottom of Table 10.

If all the members of the frame had constant moment of inertia, the final moments acting on BC and Bb would be 190 and -1350 respectively instead of 773 and -1933 foot-pounds respectively. The final moments acting on CB and BA would of course be identical for both cases.

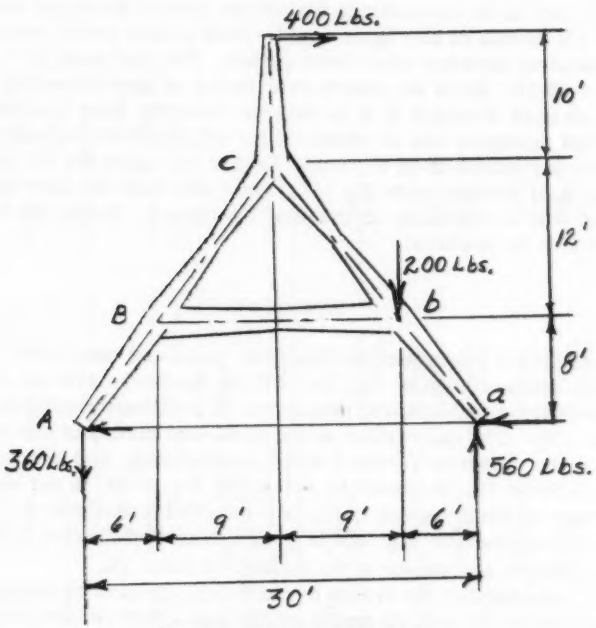
DISCUSSION AND CONCLUSION

If external loads are applied on the members or moment loads on the joints (Fig. 8), it is necessary to carry out an ordinary moment-distribution operation first, employing auxiliary forces at the joints along one chord to

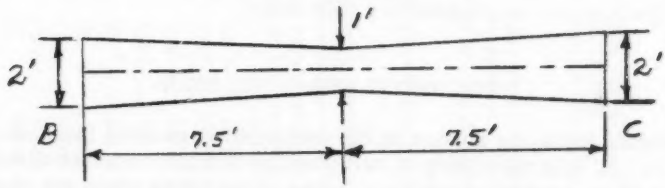
Table 7 Distribution operation for Example 2
All quantities in feet and pounds

Joint	A		B			C			D			E			F	
Member	Aa	AB	BA	Bb	BC	CB	Cc	CD	DC	Dd	DE	ED	Ee	EF	FE	Ff
C_1		-2	-.5		-1.25	-.8		-1	-1		-.8	-1.25		-.5	-2	
r	.956	.044	.147	.786	.067	.106	.808	.086	.086	.808	.106	.066	.788	.146	.041	.959
M_F		+1047	+907		+1020	+975		-1000	-1000		-650	-680		-604	-698	
		+141	-283	-1515	-129	+161		-142	+142	+1333	+175	-140				
	-1136	-52	+104								-118	+94	+1122	+208	-104	
		+8	-15	-82	-7	+9		-10	+10	+95	+13	-10		-66	+33	+769
	-8	0									-6	+5	+60	+11	-5	
					0	+1	+5	+1	0	+5	+1	0				+5
									0							
M Values from Ex. 1	-1144	+1144	+713	-1597	+884	+1146	+5	-1151	-848	+1433	-585	-731	+1182	-451	-774	+774
	-1193	+1193	+613	-1451	+838	+1202	+17	-1219	-781	-1332	-551	-759	+1125	-366	-816	+816
Difference in %	-4	-4	+16	+10	+6	-5		-5	+9	+8	+6	-4	+5	+23	-5	+5

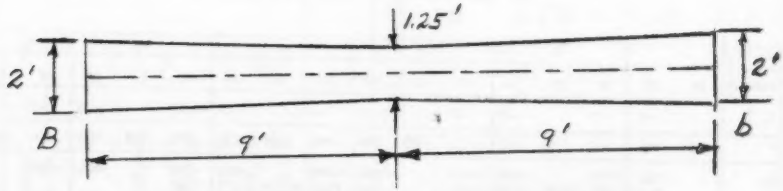
Table 6 Distribution factors for Example 2



(a) Dimensions and loads



k=k'=17.34; C=C'=.692; K_o=.0833E/15=.00556E



k=k'=10.72; C=C'=.633; K_o=.1628E/18=.00905E

(b) Dimensions and flexural properties of members

Fig. 7. Frame used in Example 4.

Member	CB	BC	BA	AB
a	∞	0	1.667	.6
M_E	+2000	-4400	+6200	-8400
M_E'	-4400	+2000	-8400	+6200
M_F	+2000	+1384	+1160	0
aM_E+M_E'	∞	+2000	+1933	+1160
aM_F+M_F'	∞	+2000	+1933	+1160
$aM+M'$	∞	+2000	+1933	+1160

Table 8 Fixed-end moments for Example 4
All quantities in feet and pounds

Joint	Member	S_1 or S_2		r
B	BC	$17.34(1 - .692 \times .692)\ .00556E$	= .0502E	.240
	Bb	$10.72(1 + .633)\ .00905E$	= .1586E	.760
	BA		0	0
			.2088E	1.000

Table 9 Distribution factors for Example 4

Joint	C	B		
Member	CB	BC	Bb	BA
r		.240	.760	0
M_F	+2000	+1384		+1160
		-611	-1933	
M	+2000	+773	-1933	+1160

Table 10 Distribution operation for Example 4
All quantities in feet and pounds

restrain the bents against sidesway.* At the end of this operation, these auxiliary forces can be computed and removed in one operation by applying equal and opposite forces at the same joints using the proposed method. The resultant forces acting in the structure due to the given loads are obtained by the superposition of the results of the two operations.

So far the discussion has been confined to Vierendeel bents which are free to deform in the manner prescribed by the "zero load moment distribution" process, i.e., Vierendeel bents in which M_E and M'_E can be determined by statics. For the case shown in Fig. 9a, M_E and M'_E cannot be calculated by statics, but a solution may be obtained by means of the superposition of the two force systems shown in Fig. 9b, c. In the solution for the force system shown in Fig. 9c, any arbitrary value of F may be used. The deflections d_1 and d_2 may be determined by any method(11) once the bendint moments along the chord members are known. The reactions and the internal forces due to the loads shown in Fig. 9a may then be obtained by the relationship

$$f_a = f_b + \frac{d_1}{d_2} f_c$$

in which f_a, f_b, and f_c respectively denote the corresponding internal or external forces in the three force systems.

The method presented in this paper employs the successive approximation technique of ordinary moment-distribution method requiring only minor modification. The expressions involved in the determination of fixed-end moments, stiffness and carry-over factors are relatively simple and the speed of convergence is rapid enough for practical cases to render it a useful and powerful tool for the analysis of Vierendeel bents. Equations (2a, b) provide simple check, to some extent, on the numerical work. It is seen that the influence of haunches, cover plates and gusset plates on the final moments is significant and can be taken in account with very little extra effort.

BIBLIOGRAPHY

1. Cross, H., "Analysis of Continuous Frames by Distributing Fixed End Moments," Trans. ASCE, Vol. 96, 1932.
2. Maugh, L. C., "Statically Indeterminate Structures," John Wiley & Sons, Inc., New York, 1946.
3. Maugh, L. C., "The Analysis of Vierendeel Trusses by Successive Approximation," Pub. Int. Ass. Bridge Struct. Engng., Vol. 3, 1935.
4. Grinter, L. E. and Tsao, C. H., "Joint Translation by Cantilever Moment Distribution," Trans. ASCE, Vol. 119, 1954, pp. 1195.
5. Lightfoot, E., "A Moment-Distribution Method for Vierendeel Bents and Girders with Inclined Chords," Proc. Instn. Civ. Engrs., Vol. 10, July, 1958, pp. 321-352.

*An exception is a moment load applied at the vertex of a triangular panel, which can be treated directly by the proposed method.

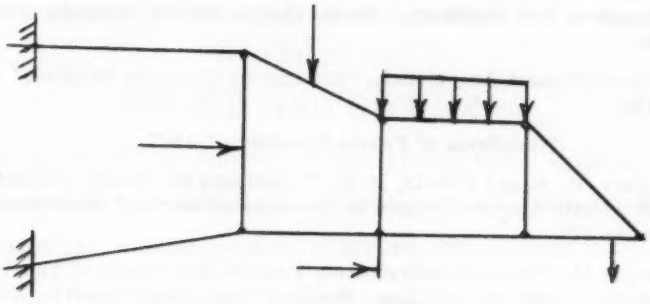


Fig. 8. Vierendeel bent subjected to loads applied on the members.

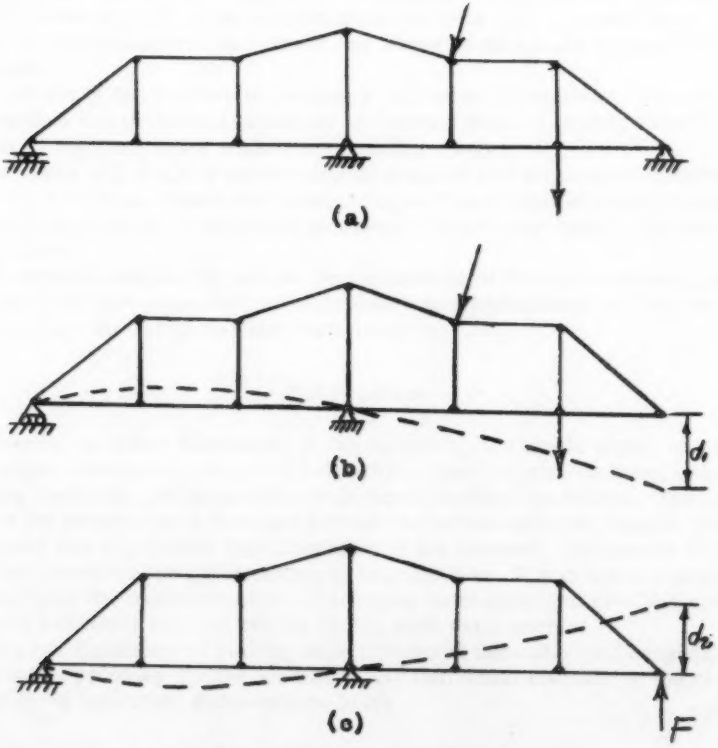


Fig. 9. Continuous Vierendeel Truss.

6. Tsao, C. H., "The Analysis of the Vierendeel Truss and of the Wind Stresses in Tall Buildings," Ph.D. Thesis, Illinois Institute of Technology, 1952.

7. Portland Cement Association, "Continuous Concrete Bridges," 2nd. Ed. ca 1951.

8. . . . . "Handbook of Frame Constants," 1947.

9. Caughey, R. A. and Cebula, R. S., "Constants for Design of Continuous Girders with Abrupt Changes in Moments of Inertia," Iowa State College Bulletin 176.

10. Gere, J. M., "Moment Distribution Factors for Beams of Tapered I-Sections," Dept. of Civil Eng., Stanford Univ., distributed by American Institute of Steel Construction.

11. Lee, S. L., "The Conjugate Frame Method and its Application in the Elastic and Plastic Theories of Structures," J. Franklin Inst., Vol. 266, No. 3, Sept. 1958, pp. 207-222.

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COMPUTER DESIGN OF A MULTISTORY FRAME BUILDING

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The exact elastic analysis of a multistory frame building of any substantial size is best described as a massive operation. What work is done is usually confined to academic studies and to the analysis of large and very important structures. Practical work has, for a large part, been directed towards the development and justification of suitable short-cuts. As a result, there are a number of approximations in current use, some of which are reliable within limitations.

The advent of the electronic computer has made it possible to reassess many routine computational problems and among these it is only natural to find that of the analysis of multistory building frames.

This paper will discuss the development and use of a computer program designed for a medium-speed electronic computer and capable of analyzing building frames of up to 100 joints in extent without overlapping of portions of the structure.

Of interest to engineers will be the byproducts of this development, particularly in the use of methods of successive approximations on large scale problems with the aid of an electronic computer.

The Problem

The problem under discussion is the analysis, in a single plane, of a multistory frame structure, subject to both vertical and lateral loadings, to provide resulting moments and shears under different loading conditions. The program in its present form does not provide for offset columns, though it has been found that in practice this limitation is not serious. To provide for this would not, however, be a difficult or costly addition. It was thought desirable to investigate the basic technique of analysis thoroughly and give it a considerable practical workout before adding such refinements.

Complete flexibility of loading was considered desirable and consequently the program provides for the loading of any individual member of the structure with up to 50 individual and separate loads.

Note: Discussion open until May 1, 1960. To extend the closing date one month, a written request must be filed with the Executive Secretary, ASCE. Paper 2322 is part of the copyrighted Journal of the Structural Division, Proceedings of the American Society of Civil Engineers, Vol. 85, No. ST 10, December, 1959.

1. Enelco Ltd., Toronto, Ont., Canada.

It was not the purpose of this program to obtain results to an accuracy beyond that required or usable in a design office. Under normal use the program was designed to achieve accuracy of 2% - 3% on sidesway loadings with, however, the proviso that additional accuracy could be obtained simply by allowing the program to continue to operate at greater length.

Fundamental considerations included the requirement that the operating time and the cost of solving a problem in this manner on the computer should be compatible with usefulness of the results obtained in a normal design office. The main purpose of the program was, therefore, of a practical rather than academic nature. However, the experience gained in the preparation of this program should be of considerable general interest.

Method of Solution

With the above goals in mind, the search for a method of solution was in progress long before actual coding began. When this problem was first approached, it was the feeling that the most applicable method would involve "matrix algebra". The use of matrix solutions on a large scale has been mentioned so often recently that a more prosaic approach by moment distribution seemed to be almost trivial in nature. Consequently, various methods including slope deflection were carefully studied. Most of these involved the solution of large sets of simultaneous equations. The use of such matrix methods appeals to mathematicians. However, the solution time of available subroutines for the solution of large sets of simultaneous equations on medium-speed equipment is such as to make the matrix approach economically questionable for the size of the problem under study.

Since it was desired to use a medium-speed computer which would be accessible and available to a normal design office, present operational times for sets of unknowns in excess of 30 are unacceptable. The solution time for matrices varies approximately as the cube of the number of unknowns, so that this avenue of approach can easily reach a serious mathematical hurdle.

It is not the purpose of this paper to discuss the various solutions of large sets of equations, but suffice it to say that what we wished was a reliable method which would be free of the danger of divergence when operating under normal conditions and in which the designer could at all times obtain a rapid appreciation of the progress of the calculations.

New mathematical methods may be developed to solve the very special types of equations encountered in the solution of a frame and of course we must always keep an open mind on such matters. There has been much misinterpretation of the principle that with an electronic computer one must review the manual methods of approach. The word "review" is the keyword and should not be misread for "abandon"; it should be interpreted rather as "revise". Consequently, one should not adopt a dogmatic position on moment distribution, since with this method we are, in fact, solving sets of simultaneous equations by an iterative procedure. In the solution of large sets of simultaneous equations the use of an iterative method of solution is itself a common procedure since elimination is not always a very attractive mathematical procedure. There are many very excellent articles and discussions on the solution of linear algebraic equations by means of iteration or relaxation. One of the fundamentals of such an approach is that the better the initial approximation for the unknowns, the more rapid the convergence to a final result. In fact, if the equations are poorly conditioned, the equations cannot

be solved at all by this method and more time-consuming procedures must be used. In our problem, what better initial approximation than fixed end moments?

Out of this emerged some interesting observations. The method of solution bears a close relationship to the tools available to the engineer. Once Hardy Cross had stated the principles of moment distribution, engineers have been evolving refinements, procedures, short-cuts, and adaptations all centered around the use of pencil, paper and slide rule. We must now return to original principles and evolve new refinements centered around the use of a magnetic surface as a calculation sheet, electrical circuitry as writing tools and electronics as an accounting device. It is just as essential that engineers become familiar with computational principles as it is for them to learn how to read, write and use the slide rule, if we are to expect the same progress in the use of our new design tool.

In general, there are two writing surfaces available to the engineer when he uses a computer. One is the magnetic drum, the other the magnetic core. Usually the magnetic drum can only be written upon or read on a cyclical basis, i.e. once or twice a revolution depending on the layout of reading and writing heads. We therefore have the problem of access time, not usually stressed by machine manufacturers, except when they are selling magnetic cores, any portion of which are available at any time. Cores, however, are still a feature of only high-speed or higher cost computers and were therefore not further considered. We were therefore left with a drum and the fundamental problem of sequencing our computations in order to minimize lost time. This rules out absolutely the use of interpretative routines, unless machine time is of no concern. Operations involving repeated iteration require optimized machine language programming to be effective. The tendency of engineers to shy away from more complex machine language programming is regrettable, since they waste the tremendous potential of the equipment and deny themselves the use of powerful short-cuts which, when considered simultaneously with the problem and design methods at hand can lead to spectacularly successful computer applications instead of the many many marginal cases which disappoint so many people almost every day.

The drum then is our fundamental writing surface and calculation sheet. The ability of the computer to operate upon this drum is a function of the individual machine. Before beginning on a problem of this nature, the engineer would be well advised to consider carefully the various commands he has at his disposal for the execution of the operations on his magnetic calculation sheet. The nature of this command list will govern his approach to the solution and enable him to devise short-cuts in his detailed programming. Due to the sequential nature of the accessibility of various portions of the memory it is important that the programmer be able to avoid any unnecessary drum revolutions for the execution of operations which may have little or no actual computational significance. Such operations are called decision operations or less politely "red tape".

Certain computers have what is referred to as "short access lines". The Bendix G-15-D computer used in this particular case has a total of 20 lines each of 108 words or slots. This provides a normal storage capacity of 2160 words. Each word is accessible once during a drum revolution. In addition, it has 4 rapid access lines each of 4 words which are accessible 27 times during a drum revolution, giving a total storage capacity of 2176.

At the beginning of every individual cycle of moment distribution and carry-over, the various applicable data and coefficients can be transferred to the short lines at the beginning of an individual computational cycle and the whole sequence of computation planned with greater efficiency. This optimization or microcoding, as it is sometimes called, may appear to be a somewhat erudite refinement. This, however, is not true. It is, rather, the core of the whole solution. It is true that an individual drum cycle takes very little time, but it is also obvious that a large number of words and a large number of figures must be handled in the course of a typical solution. For instance, the loading and data sorting operation had originally been laid out in a very casual manner with no attempt made to eliminate waste drum revolution. Initially the operation took twelve and one half minutes for a full 100 joint frame. More careful planning cut this time to two and one half minutes. When the overall machine time for solution of a problem of this nature for a 100 joint frame takes 20 minutes for a full 4 cycles of moment distribution without sidesway, it is easy to see that this time could have been as much as an hour and a half to two hours with less careful planning. Further considering that sidesway solutions normally require anywhere up to ten times the amount of machine time as that for a solution without sidesway, it can be readily seen that the time for the solution of a major frame without such optimization would have been prohibitive.

Another factor of importance in coding such a problem is the ensuring that the basic approach is sufficiently flexible to permit variations at a later date. For instance, the program as written now can be readily modified to allow for the insertion at any point of arbitrary, externally applied, moments. This means that the program in its present form could be adapted quite readily to adjacent multistory frames linked with torsion resistant members. Time spent in such foreplanning is usually time well spent.

General Flow Chart

A general flow chart for this program, which is self explanatory, is shown herewith.

Detailed Coding

In addition to the points mentioned in the previous section, there are several features in the coding of this problem which would be of interest to the engineer planning similar problems in the future. Those not familiar with coding procedures may bypass this section which is included for the benefit of the growing number of engineers familiar with detailed coding procedures.

In this program great use has been made of command modifications with the modifiers being operated on out of the short access memory. This procedure greatly reduces waste drum revolutions.

Example of Command Modification

The ability to modify commands in coding is fundamental to obtaining an economic computation speed and maintaining adequate working storage space.

BUILDING FRAME ANALYSIS
GENERAL FLOW SHEET

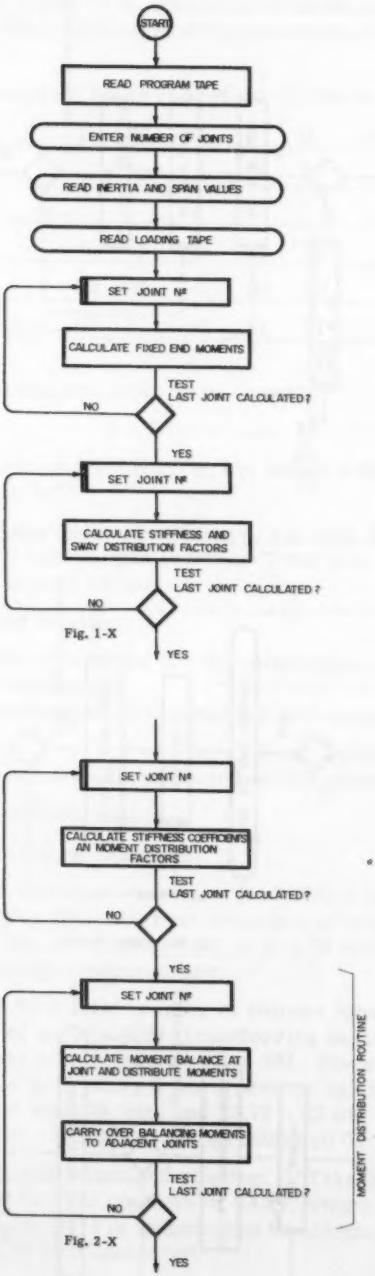


Fig. 1-X

Fig. 2-X

Fig. 3-X

Fig. 4-X

Fig. 5-X

- Location in line 09 contains I values at bottom of joints 1 to 100 inclusively in word positions 02 to 101 inclusively.

- Location 04.70 (line 04 word position 70) contains the base command, transferring word time in line 09 to accumulator (designated as line 28).

Command sequence for command modification extracting 09.xx to accumulator is as follows:

Commands being obeyed out of line 04

Step	L	P	T	N	C	S	D	BP
Storage	70		01	73	1	09	28	
Step 1	69		70	72	0	04	28	
Step 2	72		73	75	1	23	29	
Step 3	75		77	77	0	31	31	

$\frac{1}{18}$ drum revolution = $\frac{1}{540}$ sec. = 1.85 millisecs.

L = Location of command about to be obeyed.

T = Word time S = Source Line D = Destination Line

S.T. = Location of information Source, i.e. with S = 04 T = 70 Source of information is 0470.

D.T. = Location of Information Destination, i.e. with D = 04 T = 70 Destination of information is 0470. T for both Source and Destination must always be same.

N = Next command location.

C = Characteristic of transfer C = 0 = numbers + ve and - ve are transferred unchanged.
C = 1 + ve unchanged. - ve numbers are complemented during transfer.

P = normal or block transfers (unused in this example).

BP = Break Point in calculations.

Storage 04.70 contains base command.

Step 1 - Location 04.69 - Base command (in 0470) is transferred unchanged to accumulator (D = 28), next command is from (0472) since all commands are being obeyed out of line 04 until a command is inserted to change command line.

Step 2 - Location 04.72 - Joint counter in storage location 23.73 is added to contents of accumulator (transferring to line 29 automatically adds contents to accumulator (line 28). Since line 23 is short rapid access line, every 4 word position is identical, i.e. position 23.00 = 23.04 = 23.08, etc. and 23.73 = 23.01. E.g. 01.73.1.09.28 + 03.00.0.00.00 = 04.73.1.09.28 operating on 09.04.

Step 3 - This is a special command reading. ("Take next command from accumulator"). The contents of 09.xy, where xy = T (= 01 of base command + counter) is transferred to accumulator, i.e. in above example 09.04 is transferred.

Note: As computation proceeds, therefore, the command modifies itself as required to conform with the joint under consideration with virtually no lost time.

Vertical Loading

The analysis of frames under vertical loading is straightforward and, as may be seen from the flow chart, the distribution cycle proceeds in an orderly fashion from top left to bottom right of the envelope of joints. The shape and size of the structure is designated by the total number of joints in the frame and by the number of joints per floor. Irregular arrays are provided by listing non-existent members with real lengths and zero moments of inertia.

The number of moment cycles is set arbitrarily at four though a simple closure test could have been provided at the sacrifice of data storage space. If by visual inspection four cycles are insufficient, an additional four cycles can be performed quite readily. Or the number of cycles can be pre-set at any desired number.

Four cycles of moment distribution for vertical loadings are quite sufficient for most practical cases except where sway is a major factor. Sway will be discussed in the next section.

A large capacity for loading data is achieved by computing fixed end moments as the loads for each member are being read in. Thus only 100 storage spaces are required for active storage of loadings. Since two words are required to describe each load (magnitude and distance from joint at left of member), this means that each member can be loaded with up to 50 separate loads. Thus any shape or form of loading can be readily specified. Uniform loads are replaced with ten equally spaced individual loads (the program could be modified to handle U.D.L. but again space considerations governed). In large frames it has been found that repetition of loadings is quite common and given suitable tape duplication facilities the input time for multiple load substitution for U.D.L.'s is not a major factor. In fact, the procedure is simple and readily provides for partial U.D.L.'s and triangular distribution.

The moment type out is sequential in the order of joints. Typical frames (some with moment diagrams) are in Figs. 1 to 6. Corresponding computer output is shown with suffixes a to d attached to the frame number and input sheets for frame no. 1 are shown in Figs. 1e, 1f and 1g.

Once the moment typeout is complete, the operator is given the option of refining the results or proceeding to a typeout of the shears. Once these are completed, the midspan moments are computed for each joint at which a load has been applied. The method of input thus gives the designer an opportunity of selecting the positions for which he wishes moment typeouts.

Sway Correction

Sway correction is provided by a simple convergence procedure. Each cycle of computation consists of a single sway distribution and four moment distribution cycles. The overall cycle is repeated as many times as is required to provide sufficiently accurate shear values at each floor. The procedure is best explained by an example.

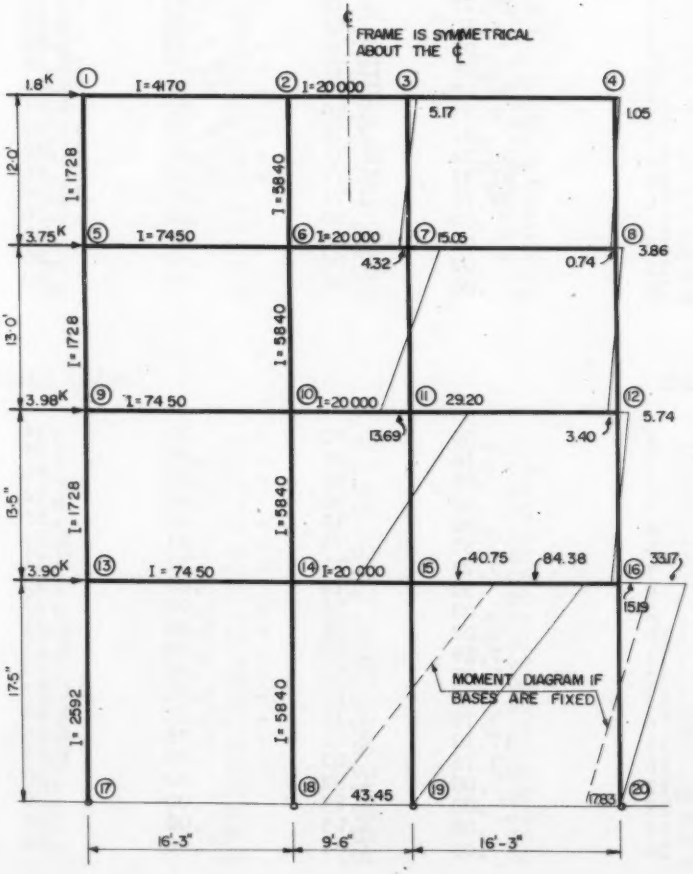
In the frame shown in Fig. 7 three applied forces of 1 k each are applied to the left of joints 1, 5 and 9 respectively. The total horizontal shear forces in levels 1, 2 and 3 are therefore 3, 2 and 1 kips respectively.

If the moments at the top and bottom of columns in each level are added and the total divided by the story height we have the horizontal shear existing in the level for those column moments.

$$\text{i.e. } \frac{M_1^4 + M_2^4 + M_3^4 + M_4^4 + M_5^2 + M_6^2 + M_7^2 + M_8^2}{12} = H_3$$

At the first computation cycle M_k^4 and M_k^3 are always zero and thus $H_n = 0$. The error is 1^k. The total correction to be applied to level 3 is therefore 1^k. This produces a total moment of 12,000 ft. lbs.

I for level 3 = 1 + 2 + 2 + 1 = 6



FRAME No. 1

FIG. No.1

Moments (ft. kips)

Joint #	Left	Top	Right	Bottom
1	.00	.00	- 1.00	1.00
2	- .76	.00	- 4.22	4.98
3	- 4.22	.00	- .76	4.97
4	- 1.00	.00	.00	1.00
5	.00	.69	- 4.56	3.86
6	- 3.94	4.12	- 15.27	15.09
7	- 15.27	4.12	- 3.95	15.09
8	- 4.56	.69	.00	3.87
9	.00	3.39	- 9.23	5.84
10	- 8.39	13.71	- 34.67	29.34
11	- 34.67	13.71	- 8.39	29.34
12	- 9.23	3.40	.00	5.84
13	.00	2.75	- 34.75	32.00
14	- 26.24	26.14	- 81.37	81.46
15	- 81.37	26.14	- 26.24	81.46
16	- 34.75	2.75	.00	32.00
17	.00	.00	.00	.00
18	.00	- .00	.00	.00
19	.00	- .00	.00	.00
20	.00	.00	.00	.00

Shears

1	.00	.00	- .10	.14
2	.10	.00	- .88	.75
3	.88	.00	- .10	.75
4	.10	.00	.00	.14
5	.00	.14	- .52	.55
6	.52	.75	- 3.21	2.21
7	3.21	.75	- .52	2.21
8	.52	.14	.00	.55
9	.00	.55	- 1.08	.63
10	1.08	2.21	- 7.29	4.11
11	7.29	2.21	- 1.08	4.11
12	1.08	.55	.00	.63
13	.00	.63	- 3.75	1.82
14	3.75	4.11	- 17.12	4.65
15	17.12	4.11	- 3.75	4.65
16	3.75	.63	.00	1.82
17	.00	1.82	.00	.00
18	.00	4.65	.00	.00
19	.00	4.65	.00	.00
20	.00	1.82	.00	.00

Frame #1—Wind Loading

Fig. 1a

Moments (ft. kips)

Joint #	Left	Top	Right	Bottom
1	.00	.00	6.40	- 6.40
2	- 16.09	.00	11.13	4.96
3	- 1.00	.00	- .09	1.09
4	- .24	.00	.00	.24
5	.00	- 7.12	14.36	- 7.24
6	- 34.81	6.27	21.76	6.78
7	- .05	.68	- .81	.19
8	- .55	.29	.00	.26
9	.00	- 7.20	14.49	- 7.28
10	- 34.87	6.70	21.30	6.87
11	.21	.26	- .69	.22
12	- .52	.26	.00	.27
13	.00	- 7.68	12.11	- 4.43
14	- 35.38	7.30	24.58	3.50
15	.55	.04	- 1.05	.47
16	- .66	.27	.00	.39
17	.00	.00	.00	.00
18	.00	.00	.00	.00
19	.00	.00	.00	.00
20	.00	.00	.00	.00

Shears (kips)

1	.00	.00	4.27	- 1.12
2	5.47	.00	3.91	.93
3	1.78	.00	- .02	.14
4	.02	.00	.00	.04
5	.00	- 1.12	9.73	- 1.11
6	12.26	.93	5.83	1.03
7	1.26	.14	- .08	.03
8	.08	.04	.00	.03
9	.00	- 1.11	9.74	- 1.10
10	12.25	1.03	5.81	1.04
11	1.28	.03	- .07	.01
12	.07	.03	.00	.03
13	.00	- 1.10	9.56	- .25
14	12.43	1.04	6.19	.20
15	.90	.01	- .10	.02
16	.10	.03	.00	.02
17	.00	- .25	.00	.00
18	.00	.20	.00	.00
19	.00	.02	.00	.00
20	.00	.02	.00	.00

Frame #1—Bases Hinged
Outer and Centre Span Loaded
(1.35 kips/ft. outer & o.75 kips/ft inner span)
Fig. 1b

Midspan Moments (ft. kips)

Joint #	Moment to right of joint under each load point		
1	- 2.93	9	- 6.57
1	2.44	9	5.70
1	6.23	9	14.40
1	8.42	9	19.49
1	9.03	9	21.01
1	8.05	9	18.95
1	5.52	9	13.38
1	1.34	9	4.09
1	- 4.37	9	- 8.64
1	- 11.66	9	- 24.94
2	- 9.27	10	- 18.54
2	- 6.09	10	- 13.69
2	- 3.46	10	- 9.51
2	- 1.36	10	- 6.01
2	.19	10	- 3.19
2	1.21	10	- 1.04
2	1.68	10	.44
2	1.61	10	1.24
2	1.00	10	1.37
2	- .15	10	.82
5	- 6.45	13	- 4.34
5	5.82	13	7.64
5	14.52	13	16.06
5	19.60	13	20.85
5	21.11	13	22.09
5	19.04	13	19.73
5	13.46	13	13.88
5	4.17	13	4.30
5	- 8.57	13	- 8.72
5	- 24.87	13	- 25.30
6	- 18.99	14	- 21.64
6	- 14.12	14	- 16.43
6	- 9.93	14	- 11.89
6	- 6.41	14	- 8.03
6	- 3.56	14	- 4.84
6	- 1.39	14	- 2.33
6	.10	14	- .49
6	.92	14	.67
6	1.07	14	1.16
6	.54	14	.97

Frame #1—Bases Hinged
Wind Load
Fig. 1c

Moments (ft. kips)

Joint #	Left	Top	Right	Bottom
1	.00	.00	- 1.01	1.01
2	- .76	.00	- 4.22	4.98
3	- 4.22	.00	- .76	4.98
4	- 1.01	.00	.00	1.01
5	.00	.72	- 4.51	3.79
6	- 3.92	4.13	- 15.29	15.08
7	- 15.29	4.13	- 3.92	15.08
8	- 4.50	.71	.00	3.79
9	.00	3.26	- 9.46	6.21
10	- 8.31	13.83	- 32.87	27.35
11	- 32.87	13.83	- 8.31	27.35
12	- 9.46	3.26	.00	6.21
13	.00	4.91	- 20.09	15.18
14	- 15.62	26.09	- 51.22	40.75
15	- 51.22	26.09	- 15.62	40.75
16	- 20.09	4.91	.00	15.19
17	.00	17.83	.00	.00
18	.00	43.45	.00	.00
19	.00	43.45	.00	.00
20	.00	17.83	.00	.00

Shears (kips)

1	.00	.00	- .10	.14
2	.10	.00	- .88	.75
3	.88	.00	- .10	.75
4	.10	.00	.00	.14
5	.00	.14	- .51	.54
6	.51	.75	- 3.21	2.22
7	3.21	.75	- .51	2.22
8	.51	.14	.00	.54
9	.00	.54	- 1.09	.82
10	1.09	2.22	- 6.91	3.95
11	6.91	2.22	- 1.09	3.95
12	1.09	.54	.00	.82
13	.00	.82	- 2.19	1.88
14	2.19	3.95	- 10.78	4.81
15	10.78	3.95	- 2.19	4.81
16	2.19	.82	.00	1.88
17	.00	1.88	.00	.00
18	.00	4.81	.00	.00
19	.00	4.81	.00	.00
20	.00	1.88	.00	.00

Frame #1—Bases Fixed
Wind Loading
Fig. 1d

FRAME No. 1

SPAN VALUES

DATA INPUT SHEET FOR BUILDING FRAME ANALYSIS

k	L_k^1	L_k^2	L_k^3	L_k^4
00	0000000	0000000	0000000	0000000
00	0000000	0000000	0000000	0000000
01	0000000	0000000	16.250000	12.000000
02	16.250000	0000000	09.500000	12.000000
03	09.500000	0000000	16.250000	12.000000
04	16.250000	0000000	0000000	12.000000
05	0000000	12.00000	16.250000	13.000000
06	16.250000	12.00000	09.500000	13.000000
07	09.500000	12.00000	16.250000	13.000000
08	16.250000	12.00000	0000000	13.000000
09	0000000	13.00000	16.250000	13.500000
10	16.250000	13.00000	09.500000	13.500000
11	09.500000	13.00000	16.250000	13.500000
12	06.250000	13.00000	0000000	13.500000
13	0000000	13.50000	16.250000	17.500000
14	16.250000	13.50000	09.500000	17.500000
15	09.500000	13.50000	16.250000	17.500000
16	16.250000	13.50000	0000000	17.500000
17	0000000	17.50000	0000000	0000000
18	0000000	17.50000	0000000	0000000
19	0000000	17.50000	0000000	0000000
20	0000000	17.50000	0000000	0000000
00	0000000	0000000	0000000	0000000
00	0000000	0000000	0000000	0000000

FIG. No. 1e

REF. No.

FRAME Nº 1

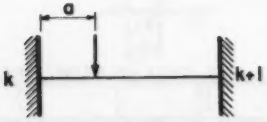
INERTIA VALUES

DATA INPUT SHEET FOR BUILDING FRAME ANALYSIS

k	I_k^1	I_k^2	I_k^3	I_k^4
1	0	0	0	0
2	0	0	0	0
3	0	0	0	0
4	0	0	0	0
5	0	0	0	0
6	0	0	0	0
7	0	0	0	0
8	0	0	0	0
9	0	0	0	0
10	0	0	0	0
11	0	0	0	0
12	0	0	0	0
13	0	0	0	0
14	0	0	0	0
15	0	0	0	0
16	0	0	0	0
17	0	0	0	0
18	0	0	0	0
19	0	0	0	0
20	0	0	0	0
0	0	0	0	0
0	0	0	0	0

FIG. No 1 f

REF. Nº



FRAME No. 1

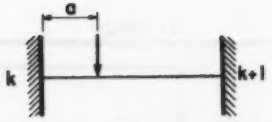
WIND LOADING

DATA INPUT SHEET FOR BUILDING FRAME ANALYSIS

k 01			k 09		
	W	0 0 0 0 0 0 0		P	0 0 0 0 0 0 0
	a or H	0 0 1·8 0 0 0		a or H	0 0 9·5 3 0 0
	P	0 0 0 0 0 0 0		P	0 0 0 0 0 0 0
	a or H	0 0 0 0 0 0 0		a or H	0 0 0 0 0 0 0
02	P	0 0 0 0 0 0 0	10	P	0 0 0 0 0 0 0
	a or H	0 0 0 0 0 0 0		a or H	0 0 0 0 0 0 0
	P	0 0 0 0 0 0 0		P	0 0 0 0 0 0 0
	a or H	0 0 0 0 0 0 0		a or H	0 0 0 0 0 0 0
03	P	0 0 0 0 0 0 0	11	P	0 0 0 0 0 0 0
	a or H	0 0 0 0 0 0 0		a or H	0 0 0 0 0 0 0
	P	0 0 0 0 0 0 0		P	0 0 0 0 0 0 0
	a or H	0 0 0 0 0 0 0		a or H	0 0 0 0 0 0 0
04	P	0 0 0 0 0 0 0	12	P	0 0 0 0 0 0 0
	a or H	0 0 0 0 0 0 0		a or H	0 0 0 0 0 0 0
	P	0 0 0 0 0 0 0		P	0 0 0 0 0 0 0
	a or H	0 0 0 0 0 0 0		a or H	0 0 0 0 0 0 0
05	P	0 0 0 0 0 0 0	13	P	0 0 0 0 0 0 0
	a or H	0 0 5·5 5 0 0		a or H	0 1 3·4 3 0 0
	P	0 0 0 0 0 0 0		P	0 0 0 0 0 0 0
	a or H	0 0 0 0 0 0 0		a or H	0 0 0 0 0 0 0
06	P	0 0 0 0 0 0 0	14	P	0 0 0 0 0 0 0
	a or H	0 0 0 0 0 0 0		a or H	0 0 0 0 0 0 0
	P	0 0 0 0 0 0 0		P	0 0 0 0 0 0 0
	a or H	0 0 0 0 0 0 0		a or H	0 0 0 0 0 0 0
07	P	0 0 0 0 0 0 0	15	P	0 0 0 0 0 0 0
	a or H	0 0 0 0 0 0 0		a or H	0 0 0 0 0 0 0
	P	0 0 0 0 0 0 0		P	0 0 0 0 0 0 0
	a or H	0 0 0 0 0 0 0		a or H	0 0 0 0 0 0 0
08	P	0 0 0 0 0 0 0	16	P	0 0 0 0 0 0 0
	a or H	0 0 0 0 0 0 0		a or H	0 0 0 0 0 0 0
	P	0 0 0 0 0 0 0		P	0 0 0 0 0 0 0
	a or H	0 0 0 0 0 0 0		a or H	0 0 0 0 0 0 0

FIG. No. 1g - 1

REF No.



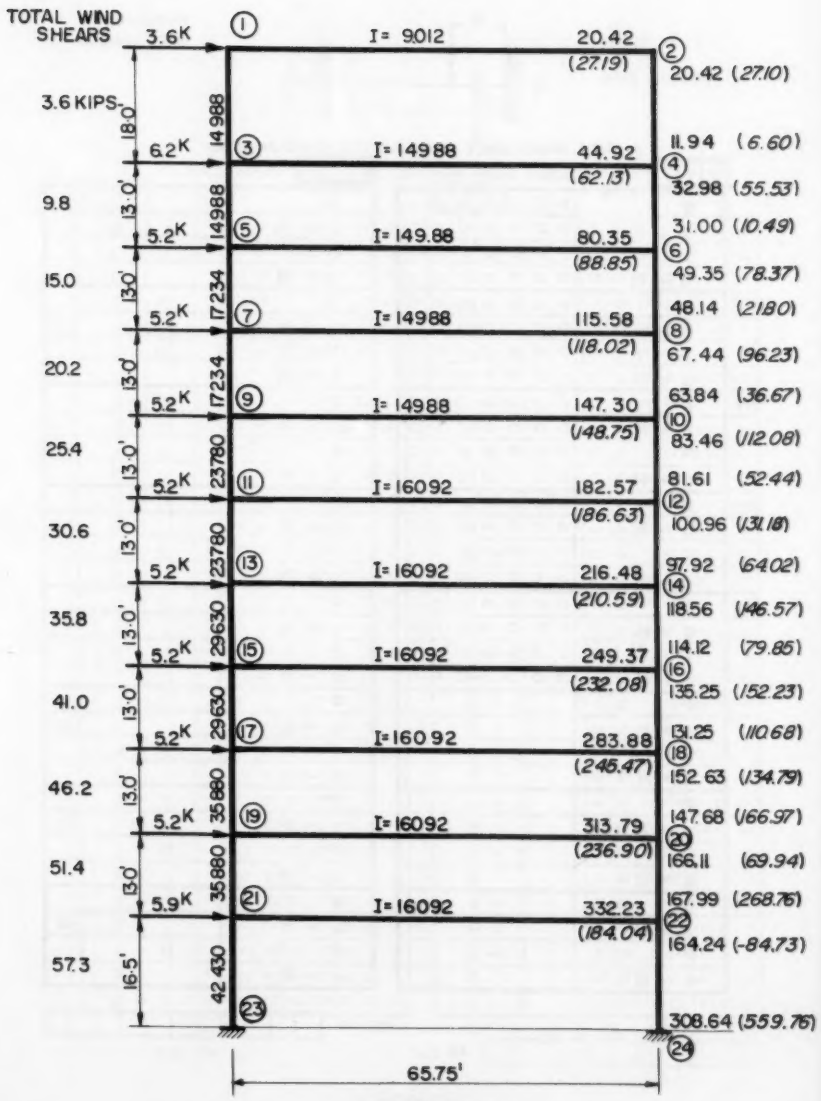
FRAME No. 1
WIND LOADING

DATA INPUT SHEET FOR BUILDING FRAME ANALYSIS

k 1 7								k							
	W	0	0	0	0	0	0	0	P						
	a or H	0	0	0	0	0	0	0	a or H						
	P	0	0	0	0	0	0	0	P						
	a or H	0	0	0	0	0	0	0	a or H						
18	P	0	0	0	0	0	0	0	P						
	a or H	0	0	0	0	0	0	0	a or H						
	P	0	0	0	0	0	0	0	P						
	a or H	0	0	0	0	0	0	0	a or H						
19	P	0	0	0	0	0	0	0	P						
	a or H	0	0	0	0	0	0	0	a or H						
	P	0	0	0	0	0	0	0	P						
	a or H	0	0	0	0	0	0	0	a or H						
20	P	0	0	0	0	0	0	0	P						
	a or H	0	0	0	0	0	0	0	a or H						
	P	0	0	0	0	0	0	0	P						
	a or H	0	0	0	0	0	0	0	a or H						
21	P	0	0	0	0	0	0	0	P						
	a or H	0	0	0	0	0	0	0	a or H						
	P	0	0	0	0	0	0	0	P						
	a or H	0	0	0	0	0	0	0	a or H						
	P								P						
	a or H								a or H						
	P								P						
	a or H								a or H						
	P								P						
	a or H								a or H						
	P								P						
	a or H								a or H						
	P								P						
	a or H								a or H						

FIG. No. 1 g - 3.

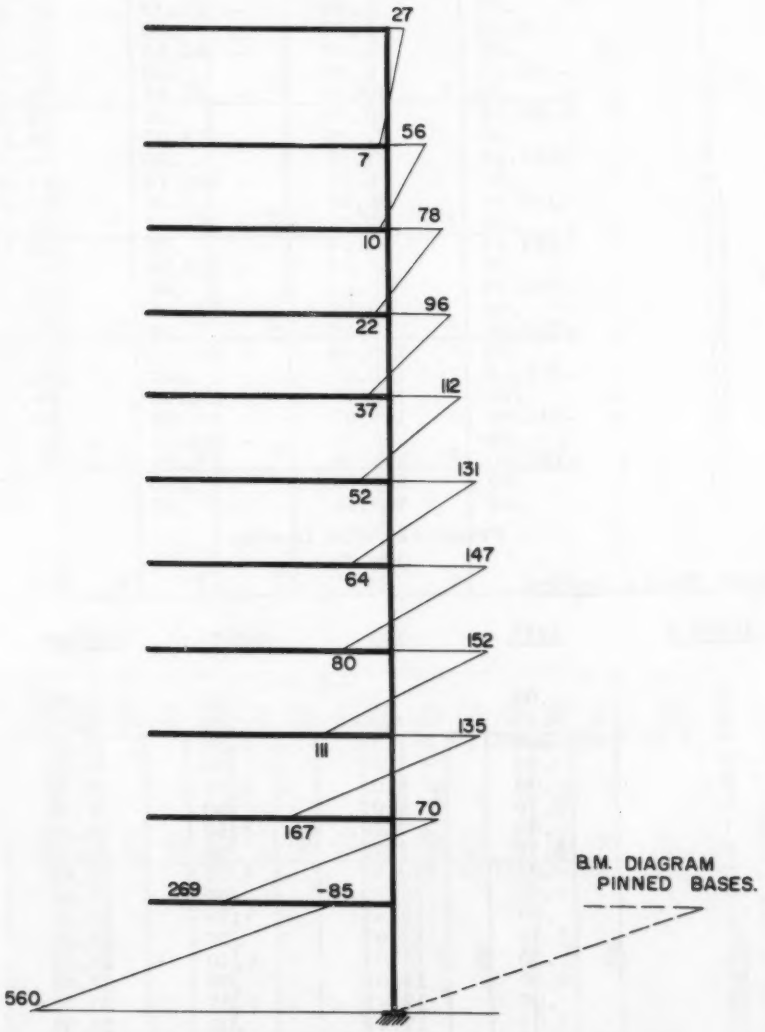
REF No.



FRAME No. 2

JOINT MOMENTS. FOR FRAME OBTAINED BY FACTOR METHOD.
RESULT GIVEN BY ELECTRONIC COMPUTER SHOWN IN BRACKETS.

FIG. No. 2



FRAME No. 2
MOMENT DIAGRAM FOR COLUMNS
FIG. No. 2a

Joint Moments in Foot Kips

Joint #	Left	Top	Right	Bottom
1	.00	.00	- 27.19	27.19
2	- 27.19	.00	.00	27.19
3	.00	6.60	- 62.13	55.53
4	- 62.13	6.60	.00	55.53
5	.00	10.49	- 88.85	78.37
6	- 88.85	10.49	.00	78.37
7	.00	21.80	- 118.02	96.23
8	-118.02	21.80	.00	96.23
9	.00	36.67	- 148.75	112.08
10	-148.76	36.67	.00	112.08
11	.00	52.44	- 183.63	131.18
12	-183.63	52.44	.00	131.18
13	.00	64.02	- 210.59	146.56
14	-210.59	64.02	.00	146.57
15	.00	79.85	- 232.08	152.23
16	-232.08	79.85	.00	152.23
17	.00	110.68	- 245.47	134.79
18	-245.47	110.68	.00	134.79
19	.00	166.97	- 236.90	69.94
20	-236.90	166.97	.00	69.94
21	.00	268.76	- 184.04	- 84.73
22	-184.04	268.76	.00	- 84.73
23	.00	559.76	.00	.00
24	.00	559.76	.00	.00

Frame #2—Wind Loading
Fig. 2b

Joint Shears in Kips

Joint #	Left	Top	Right	Bottom
1	.00	.00	- .82	1.87
2	.82	.00	.00	1.87
3	.00	1.87	- 1.89	5.07
4	1.89	1.87	.00	5.07
5	.00	5.07	- 2.70	7.70
6	2.70	5.07	.00	7.70
7	.00	7.70	- 3.59	10.22
8	3.59	7.70	.00	10.22
9	.00	10.22	- 4.52	12.65
10	4.52	10.22	.00	12.65
11	.00	12.65	- 5.58	15.01
12	5.58	12.65	.00	15.01
13	.00	15.01	- 6.40	17.41
14	6.40	15.01	.00	17.41
15	.00	17.41	- 7.05	20.22
16	7.05	17.41	.00	20.22
17	.00	20.22	- 7.46	23.21
18	7.46	20.22	.00	23.21
19	.00	23.21	- 7.20	26.05
20	7.20	23.21	.00	26.05
21	.00	26.05	- 5.59	28.78
22	5.59	26.05	.00	28.78
23	.00	28.78	.00	.00
24	.00	28.78	.00	.00

Frame #2—Wind Loading
Fig. 2c

FRAME No. 3
FIG. No. 3
FRAME No. 4
FOR PLOT OF MOMENTS IN LOWER 5 STORIES
SEE FIGURE No. 6
FIG. No. 4
ACCUMULATED WIND SHEARS

Moments in Foot Kips

Joint #	Left	Top	Right	Bottom
1	.00	.00	- 6.06	6.06
2	- 6.30	.00	- 6.36	12.66
3	- 6.44	.00	- 5.11	11.56
4	- 4.89	.00	.00	4.89
5	.00	- 1.07	- 17.21	18.29
6	- 13.26	15.00	- 37.28	35.53
7	- 38.08	13.80	- 11.11	35.39
8	- 14.41	- 3.05	.00	17.45
9	.00	10.36	- 32.91	22.55
10	- 30.71	24.62	- 30.82	36.91
11	- 30.48	25.36	- 33.27	38.39
12	- 35.68	10.89	.00	24.79
13	.00	16.02	- 41.95	25.92
14	- 39.68	30.51	- 40.87	50.04
15	- 40.81	31.42	- 44.00	53.39
16	- 46.25	18.12	.00	28.14
17	.00	20.37	- 49.79	29.41
18	- 47.63	42.86	- 49.87	54.64
19	- 50.00	45.23	- 53.16	57.92
20	- 54.96	22.84	.00	32.12
21	.00	31.77	- 65.66	33.90
22	- 62.15	58.40	- 66.12	69.87
23	- 65.97	62.53	- 74.32	77.76
24	- 78.02	34.45	.00	43.56
25	.00	31.79	- 69.74	37.95
26	- 66.95	65.25	- 72.30	74.01
27	- 72.10	72.83	- 80.86	80.13
28	- 84.58	39.43	.00	45.15
29	.00	43.91	- 91.19	47.28
30	- 85.46	84.33	- 89.20	90.32
31	- 90.40	89.00	- 101.76	103.15
32	- 107.45	52.21	.00	55.24
33	.00	44.88	- 95.28	50.39
34	- 89.24	88.07	- 94.12	95.28
35	- 96.41	97.91	- 108.09	106.59
36	- 112.67	52.94	.00	59.73

(cont'd)

Frame #4—Wind Loading
Fig. 4a

Moments in Foot Kips

Joint #	Left	Top	Right	Bottom
37	.00	55.80	- 111.51	55.71
38	- 103.58	101.77	- 110.79	112.60
39	- 112.64	116.78	- 128.81	124.67
40	- 136.63	65.86	.00	70.77
41	.00	54.08	- 115.81	61.72
42	- 108.90	108.65	- 117.39	117.64
43	- 118.64	121.15	- 134.86	132.35
44	- 143.05	67.08	.00	75.98
45	.00	61.65	- 129.51	67.85
46	- 120.97	118.84	- 128.73	130.85
47	- 130.53	133.04	- 148.10	145.59
48	- 157.39	76.24	.00	81.15
49	.00	66.15	- 133.07	66.92
50	- 125.03	127.84	- 133.69	130.88
51	- 135.31	142.37	- 152.08	145.02
52	- 160.52	79.79	.00	80.73
53	.00	77.34	- 153.17	75.84
54	- 140.13	146.41	- 153.01	146.73
55	- 156.23	162.25	- 172.57	166.55
56	- 186.17	93.10	.00	93.07
57	.00	75.60	- 155.93	80.34
58	- 145.15	142.71	- 160.98	163.42
59	- 163.58	162.65	- 177.74	178.67
60	- 189.81	91.91	.00	97.90
61	.00	66.40	- 191.25	124.84
62	- 166.55	166.77	- 180.41	180.19
63	- 190.32	172.04	- 218.56	236.84
64	- 239.92	79.80	.00	160.12
65	.00	205.51	.00	.00
66	.00	233.18	.00	.00
67	.00	327.98	.00	.00
68	.00	268.95	.00	.00

Frame #4—Wind Loading

Fig. 4a - 1

Shears in Kips

Joint #	Left	Top		Right	Bottom
1	.00	.00	-	1.42	.39
2	1.42	.00	-	.73	2.18
3	.73	.00	-	.90	2.00
4	.90	.00		.00	.14
5	.00	.39	-	3.51	1.79
6	3.51	2.18	-	4.34	3.75
7	4.34	2.00	-	2.31	3.79
8	2.31	.14		.00	1.77
9	.00	1.79	-	7.33	3.04
10	7.33	3.75	-	3.53	5.32
11	3.53	3.79	-	6.26	5.50
12	6.26	1.77		.00	3.38
13	.00	3.04	-	9.41	3.65
14	9.41	5.32	-	4.71	7.33
15	4.71	5.50	-	8.20	7.78
16	8.20	3.38		.00	4.02
17	.00	3.65	-	11.23	4.82
18	11.23	7.33	-	5.76	8.92
19	5.76	7.78	-	9.82	9.50
20	9.82	4.02		.00	5.25
21	.00	4.82	-	14.74	5.18
22	14.74	8.92	-	7.62	10.66
23	7.62	9.50	-	13.84	11.88
24	13.84	5.25		.00	6.55
25	.00	5.18	-	15.76	6.46
26	15.76	10.66	-	8.33	12.49
27	8.33	11.88	-	15.04	13.34
28	15.04	6.55		.00	7.68
29	.00	6.46	-	20.37	7.27
30	20.37	12.49	-	10.36	14.08
31	10.36	13.34	-	19.01	15.86
32	19.01	7.68		.00	8.53
33	.00	7.27	-	21.28	8.38
34	21.28	14.08	-	10.99	15.55
35	10.99	15.86	-	20.06	17.63
36	20.06	8.53		.00	9.91

(cont'd)

Frame #4—Wind Loading

Fig. 4b

Shears in Kips (cont'd)

Joint #	Left	Top	Right	Bottom
37	.00	8.38	- 24.80	8.66
38	24.80	15.55	- 12.89	17.46
39	12.89	17.63	- 24.13	19.40
40	24.13	9.91	.00	10.88
41	.00	8.66	- 25.91	9.73
42	25.91	17.46	- 13.61	18.66
43	13.61	19.40	- 25.26	20.94
44	25.26	10.88	.00	12.01
45	.00	9.73	- 28.89	10.57
46	28.89	18.66	- 14.96	20.41
47	14.96	20.94	- 27.77	22.72
48	27.77	12.01	.00	12.70
49	.00	10.57	- 29.77	11.38
50	29.77	20.41	- 15.52	21.88
51	15.52	22.72	- 28.41	24.25
52	28.41	12.70	.00	13.71
53	.00	11.38	- 33.82	11.95
54	33.82	21.88	- 17.84	22.84
55	17.84	24.25	- 32.61	25.98
56	32.61	13.71	.00	14.59
57	.00	11.95	- 34.72	11.58
58	34.72	22.84	- 18.72	26.06
59	18.72	25.98	- 33.41	27.68
60	33.41	14.59	.00	14.02
61	.00	11.58	- 41.26	15.73
62	41.26	26.06	- 21.39	19.68
63	21.39	27.68	- 41.68	26.89
64	41.68	14.02	.00	20.43
65	.00	15.73	.00	.00
66	.00	19.68	.00	.00
67	.00	26.89	.00	.00
68	.00	20.43	.00	.00

Frame #4—Wind Loading

Fig. 4b - 1

Therefore correction moments to be applied are (in proportion to I values)

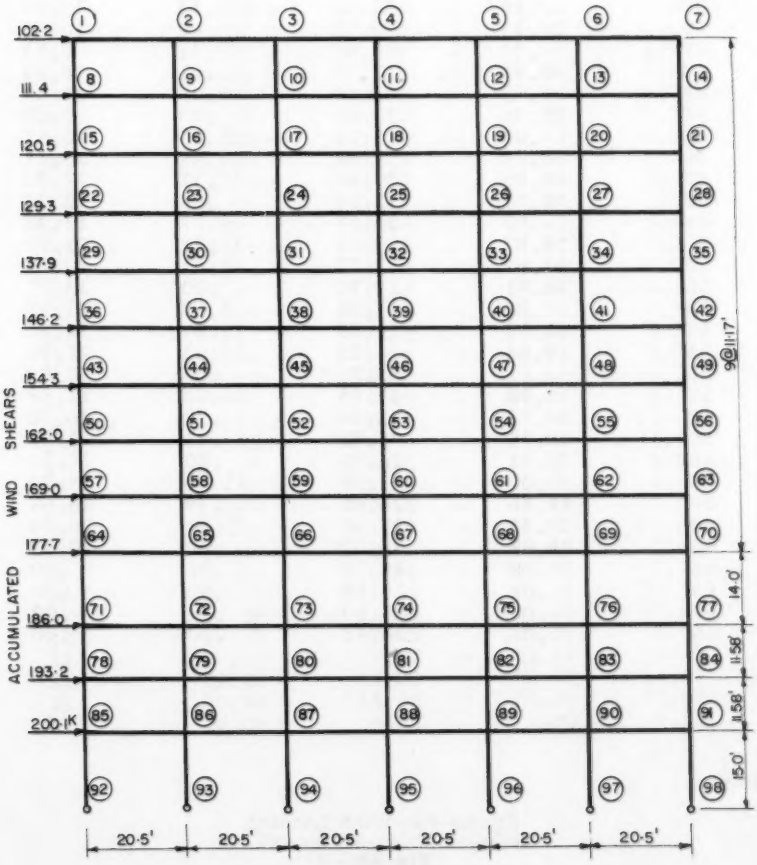
1000 ft. lb. M_1^4, 2000 ft. lb. M_2^4, 2000 ft. lb. M_3^4

1000 ft. lb. M_4^4, 1000 ft. lb. M_5^2, 2000 ft. lb M_6^2,

2000 ft. lb. M_7^2 and 1000 ft. lb. M_8^2.

After the moment distribution cycle much of this applied moment will vanish and a new error usually less than the original will have to be corrected in the same manner.

The procedure, as outlined above, converges quite slowly. Obviously, if it were known by how much the applied moments would be reduced, it would be possible initially to introduce sufficiently large correction moments, so that



FRAME No. 5

FIG. No. 5

JOINT MOMENTS IN FT. KIPS

1	.00	.00 - 67.62	67.63
2	- 65.74	.00 - 65.46	131.21
3	- 65.49	.00 - 65.52	131.01
4	- 65.51	.00 - 65.51	131.04
5	- 65.52	.00 - 65.49	131.02
6	- 65.46	.00 - 65.74	131.22
7	- 67.62	.00 .00	67.62
8	.00	17.54 - 89.48	71.92
9	- 88.64	56.32 - 89.90	122.19
10	- 89.86	56.74 - 89.82	122.92
11	- 89.82	56.73 - 89.82	122.90
12	- 89.82	56.74 - 89.86	122.92
13	- 89.89	56.32 - 88.64	122.18
14	- 89.48	17.54 .00	71.92
15	.00	35.51 - 104.13	68.64
16	- 102.79	79.47 - 103.91	127.25
17	- 103.91	79.90 - 103.90	127.92
18	- 103.90	79.90 - 103.90	127.92
19	- 103.90	79.90 - 103.91	127.92
20	- 103.91	79.46 - 102.79	127.25
21	- 104.13	35.52 .00	68.64
22	.00	44.40 - 114.23	69.85
23	- 112.89	95.62 - 114.21	131.50
24	- 114.14	96.83 - 114.08	131.42
25	- 114.08	96.81 - 114.08	131.38
26	- 114.08	96.83 - 114.14	131.42
27	- 114.21	95.62 - 112.89	131.50
28	- 114.23	44.40 .00	69.84
29	.00	43.09 - 123.07	79.96
30	- 121.63	99.65 - 123.03	144.99
31	- 122.93	100.55 - 122.83	145.18
32	- 122.83	100.52 - 122.83	145.12
33	- 122.83	100.56 - 122.93	145.16
34	- 123.03	99.65 - 121.64	144.99
35	- 123.07	43.09 .00	79.96
36	.00	51.47 - 132.71	81.25
37	- 131.37	108.75 - 133.14	155.77
38	- 133.06	109.37 - 132.99	156.68
39	- 132.99	109.29 - 132.99	156.69
40	- 132.99	109.33 - 133.06	156.73
41	- 133.14	108.75 - 131.37	155.76
42	- 132.71	51.47 .00	81.25
43	.00	57.11 - 140.01	82.96
44	- 138.81	122.73 - 140.90	157.05
45	- 140.84	123.92 - 140.76	157.74
46	- 140.75	124.08 - 140.74	157.48

(cont'd)

Fig. 5a

47	- 140.74	124.11	- 140.82	157.51
48	- 140.89	122.72	- 138.81	157.05
49	- 140.01	57.11	.00	82.95
50	.00	52.63	- 148.81	96.13
51	- 147.41	120.43	- 149.50	176.43
52	- 149.43	121.58	- 149.40	177.20
53	- 149.44	120.77	- 149.47	178.09
54	- 149.47	120.80	- 149.50	178.12
55	- 149.53	120.45	- 147.41	176.44
56	- 148.81	52.62	.00	96.13
57	.00	52.27	- 162.04	109.77
58	- 160.87	116.70	- 163.50	207.68
59	- 163.41	118.44	- 163.18	208.16
60	- 163.04	121.59	- 162.90	204.35
61	- 162.90	121.63	- 163.13	204.40
62	- 163.37	116.67	- 160.88	207.58
63	- 162.05	52.28	.00	109.78
64	.00	44.97	- 178.21	133.32
65	- 176.91	121.96	- 179.78	234.84
66	- 179.67	124.44	- 179.32	234.64
67	- 179.07	126.79	- 178.82	231.19
68	- 178.82	126.84	- 179.18	231.25
69	- 179.54	121.85	- 176.92	234.71
70	- 178.22	44.97	.00	133.32
71	.00	77.02	- 195.65	118.57
72	- 194.04	162.49	- 197.00	228.47
73	- 196.87	163.91	- 196.41	229.29
74	- 196.08	165.41	- 195.75	226.34
75	- 195.74	165.47	- 196.21	226.40
76	- 196.68	162.33	- 194.06	228.32
77	- 195.66	77.01	.00	118.56
78	.00	41.05	- 216.31	175.30
79	- 215.17	116.30	- 219.15	318.08
80	- 219.06	118.76	- 218.93	319.28
81	- 218.90	117.71	- 218.87	320.11
82	- 218.87	117.75	- 218.99	320.17
83	- 219.12	116.24	- 215.17	318.10
84	- 216.31	41.06	.00	175.31
85	.00	- 21.31	- 259.32	280.60
86	- 256.54	88.10	- 259.80	428.22
87	- 259.64	93.76	- 259.49	425.35
88	- 259.50	93.89	- 259.51	425.11
89	- 259.51	93.98	- 259.66	425.16
90	- 259.81	88.11	- 256.54	428.22
91	- 259.31	- 21.31	.00	280.60
92	.00	.00	.00	.00
93	.00	.00	.00	.00
94	.00	.00	.00	.00
95	.00	.00	.00	.00
96	.00	.00	.00	.00
97	.00	.00	.00	.00
98	.00	.00	.00	.00

Frame #5—Wind Loading

Fig. 5a - 1

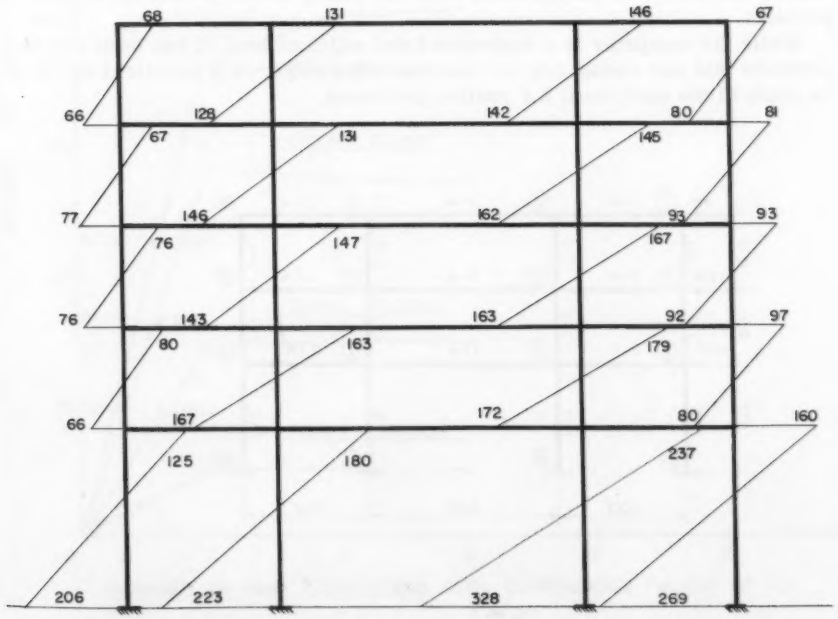
the residual M after distribution would be correct. Such a procedure is simple on single story structures, but is not practical on multistory buildings. However, the principle of overcorrection can be applied and the applied correction fixed end moments described above are multiplied by an overcompensation factor C. This factor may be any value but it has been found that a value of 1.5 is safe under practically all conditions and provides reasonably rapid closure except in the most difficult cases. A discussion on convergence of sway corrections in a typical frame follows under a separate heading.

As the number of correct significant figures increases, the solution time increases until we approach that required to solve a corresponding set of linear simultaneous equations. It is, however, significant that this technique, apart from starting with a good set of initial approximations, permits the operator to assess whether the accuracy reached at any stage is sufficient for his purposes.

Convergence of Sway Corrections

The use of this program has enabled a study to be made of the convergence of successive approximations in a structural problem. In particular can be noted the effects on rapidity of convergence of frame characteristics and the overcompensation factor used in the analysis.

Figs. 8 through 10 show the effect of various values of the overcompensation value and the effect of fixing the bases.



FRAME No. 4
DETAILED MOMENTS IN LOWER 5 STORIES. FOR FULL FRAME DIMENSIONS SEE FIG. No. 4

FIG. No. 6

It may be seen that the increase of stiffness of the columns decreases the rate of closure (with a factor of 1.5). Studies on other structures show that a further increase of column stiffness beyond that shown will begin to sharply increase the number of corrections required. In the case of infinitely stiff columns (the flagpole analogy) the closure rate is quite unsatisfactory. It may be noted that the matrix corresponding to a slope deflection solution of the same case itself presents major difficulties due to its being very badly conditioned.

Fixation of the base produces a far more rapid closure of the results than if the bases are hinged, virtual closure being achieved on the fourth cycle (with C = 1.5) as compared to the tenth for a pinned base.

Also of interest is that a C of 1.5 is more effective than 1 in this case, yet a factor of 3.2 is only an improvement if it is used for the third and fourth cycles. Beyond that the results diverge.

The obvious conclusion that can be reached is that large overcompensation factors may be used only if followed by a number of normalizing cycles with C = 1 or 1.5. It is also obvious that we can only be sure that the results are close to what we wish if:

1) A couple of cycles with a factor of 1 produce less and less change, and
2) The shear values derived from these moments are correct.

A sequence of overcompensation followed by a shear check can be readily added to the program and this will be done once studies and experience have shown the most expedient values for general use.

Overcompensation is vital to efficient use of the computer and it may be readily seen that the computation time can easily be cut in half for a given problem.

While the computer is a high-speed and efficient tool, it has been our experience that one cannot neglect machine time required if practical use is to be made of the equipment for routine problems.

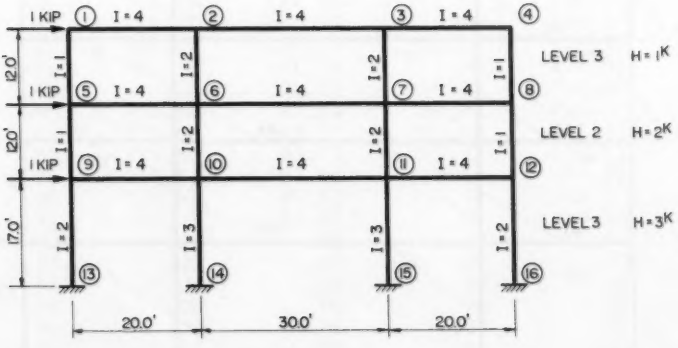


FIG. No. 7

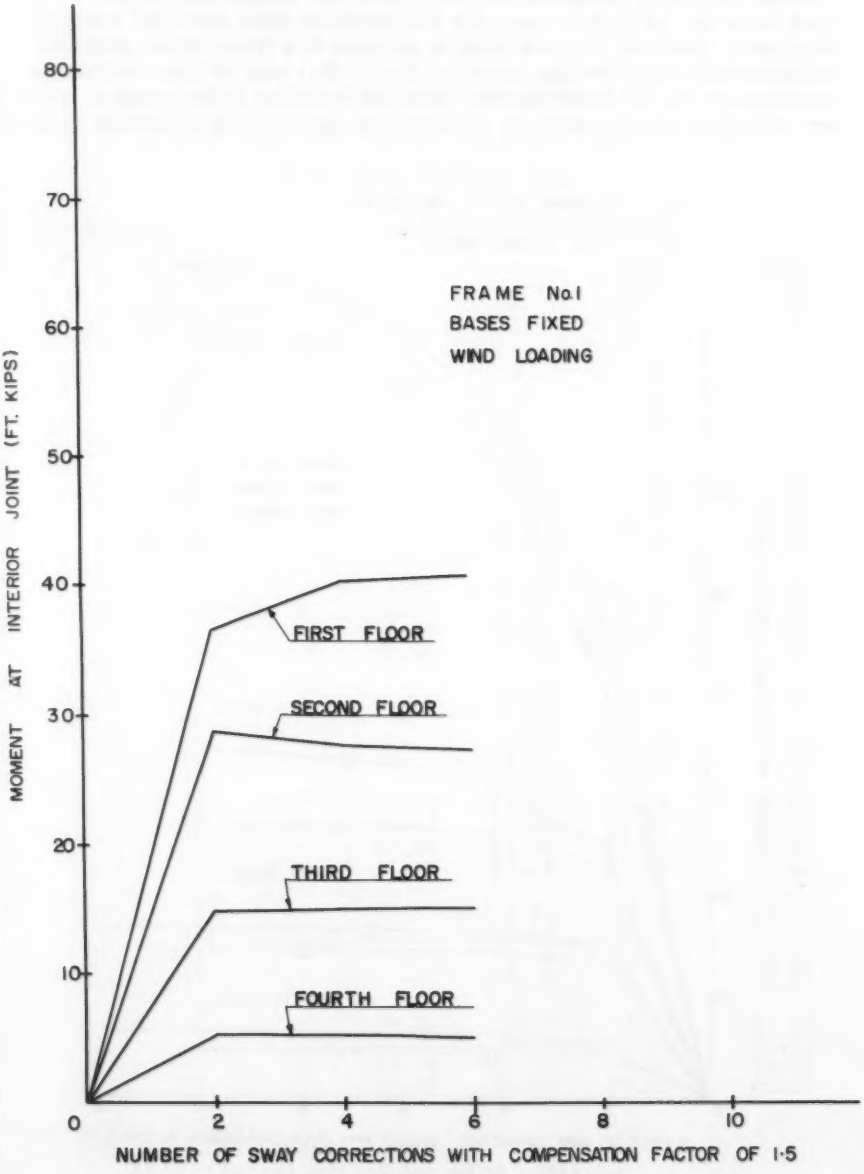

FRAME No.1
BASES FIXED
WIND LOADING
MOMENT AT INTERIOR JOINT (FT. KIPS)
80
70
60
50
40
30
20
10
0
2
4
6
8
10
FIRST FLOOR
SECOND FLOOR
THIRD FLOOR
FOURTH FLOOR
NUMBER OF SWAY CORRECTIONS WITH COMPENSATION FACTOR OF 1·5

FIG. No. 8

Comparison with Manual Approximations

The typical hospital frame in Fig. 1 has been analyzed by computer and the results checked by the portal method, cantilever method and the factor method (see Table 1). As may be seen, the last produces quite excellent results in this case. However, we have found in the case of a frame which produces column moments of the type shown in Fig. 11 that none of these methods is satisfactory for the lower stories, since the locations of the points of inflection are widely at variance with the assumptions used in the approximate methods.

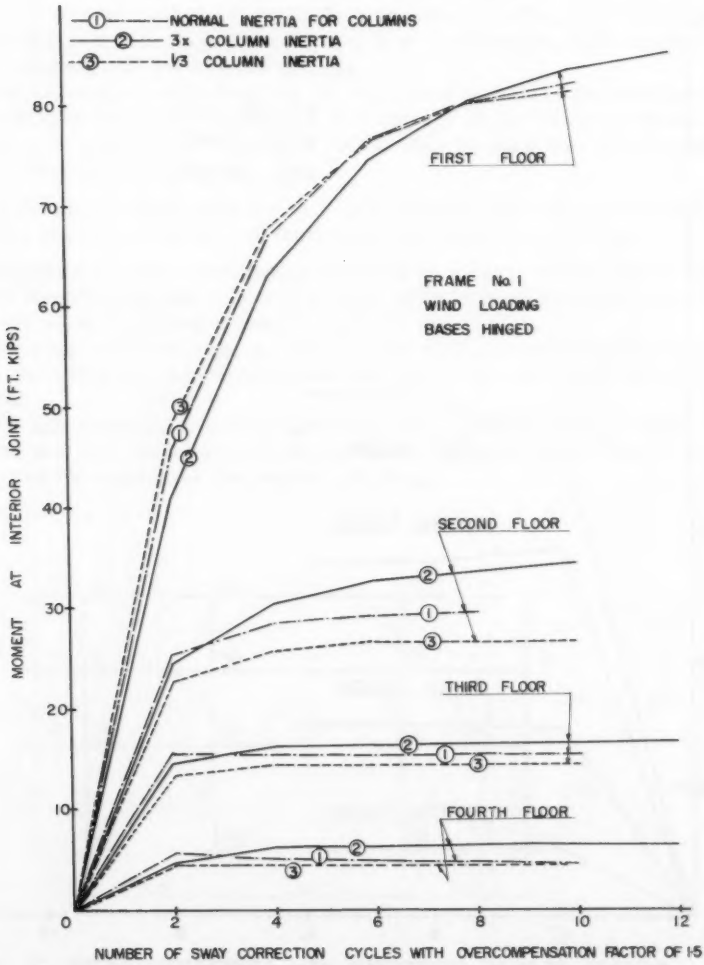


FIG. No. 9

TABLE # 1

Frame # 1

Frame subject to Wind Load

(For dimensions & properties see fig. 1)

Joint Moments in Foot Kips

Joint No.		Left	Top	Right	Bottom
1	Computer	-	-	1.05	1.05
	Portal			2.09	2.09
	Cantilever			1.93	1.93
	Factor	-	-	1.04	1.04
2	I	0.79	-	4.38	5.17
	II	2.09	-	1.22	3.31
	III	1.93	-	1.52	3.45
	IV	0.65	-	3.81	4.46
5	I	-	0.74	4.60	3.86
	II	-	2.09	9.07	6.98
	III	-	1.93	8.39	6.45
	IV	-	1.04	4.68	3.64
6	I	3.98	4.32	15.39	15.05
	II	9.07	3.31	5.30	11.06
	III	8.39	3.45	6.57	11.50
	IV	3.66	4.27	14.94	14.33
9	I	-	3.40	9.14	5.74
	II	-	6.98	19.42	12.44
	III	-	6.45	17.96	11.51
	IV	-	3.66	10.04	6.36
10	I	8.33	13.69	34.55	29.20
	II	19.42	11.06	11.36	19.72
	III	17.96	11.50	14.06	20.51
	IV	7.90	14.47	32.17	25.60
13	I	-	2.53	35.70	33.17
	II	-	12.44	57.91	45.47
	III	-	11.51	55.91	44.40
	IV	-	6.32	37.34	30.98
14	I	26.92	25.85	83.32	84.38
	II	57.91	19.72	33.86	72.05
	III	55.91	20.51	38.20	73.60
	IV	23.28	25.84	89.13	86.58

Computation Methods

Type I	Electronic computer
II	Portal method for wind analysis
III	Cantilever method for wind analysis
IV	Factor method for wind analysis

Column Shortening and Plastic Analysis

The program at present does not provide for column shortening or plastic analysis. An extension to cover both of these is theoretically possible, but as yet not fully explored.

Economics of Electronic Computation of Frames

Preparation of data and computation times will vary widely depending on the actual case, volume of work and experience of the operator. However, the typical solution of a small 20 joint frame for 5 load cases will run about as follows:

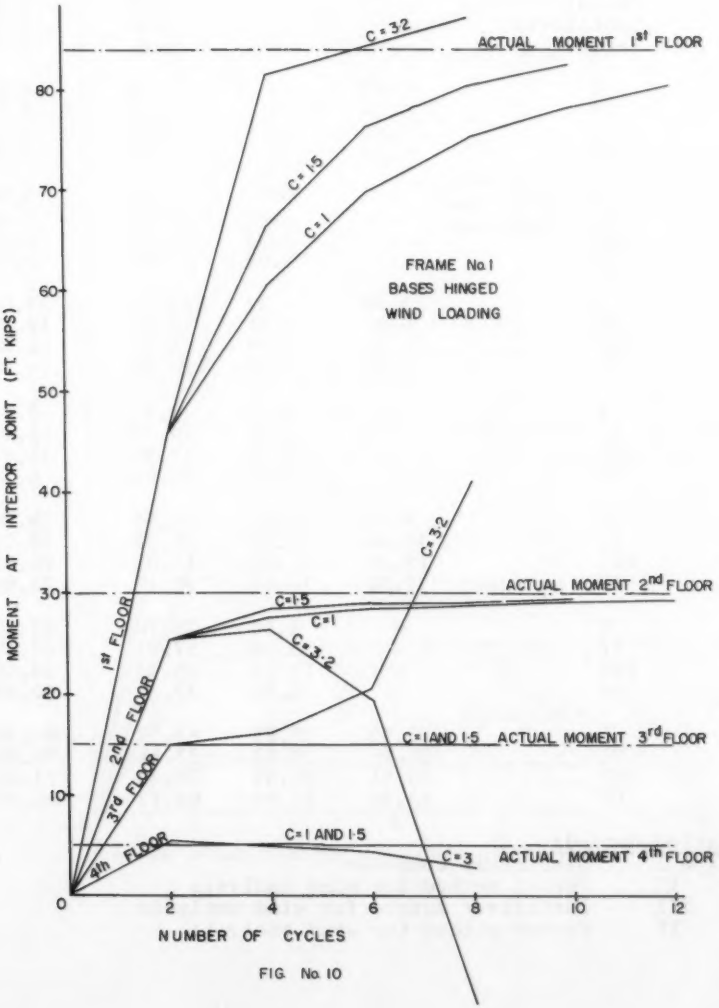


FIG. No. 10

Data Preparation

Prepare and check data sheets	2-3 man hours-engineer's time
Prepare and verify tapes	4-5 man hours-clerical only

Machine Time

Basic input time and calculation of coefficients	10 min.
4 moment distribution cycles	3 min.
Typeout of moments	3 min.
Shear computation	2 - 3 min.
Shear typeout	3 min.
Midspan moments (mostly typeout)	4 - 5 min.

Each additional sway cycle and corresponding moment distribution takes another 3 minutes each without typeout. Ten sway cycles are usually sufficient for all purposes and a judicious selection of overcompensation factors will cut this to 5 or 6. The total elapsed machine time, therefore, for the practical design of a 20 joint frame subject to 4 vertical and one lateral loading case would be 27 minutes for each vertical loading case and 37 to 50 minutes for the lateral load case which, of course, does not require the midspan moment routine. An outside figure of three hours of time on a medium-speed computer is therefore sufficient to provide the complete solution.

Extension of Computer Technique to Selection of Members

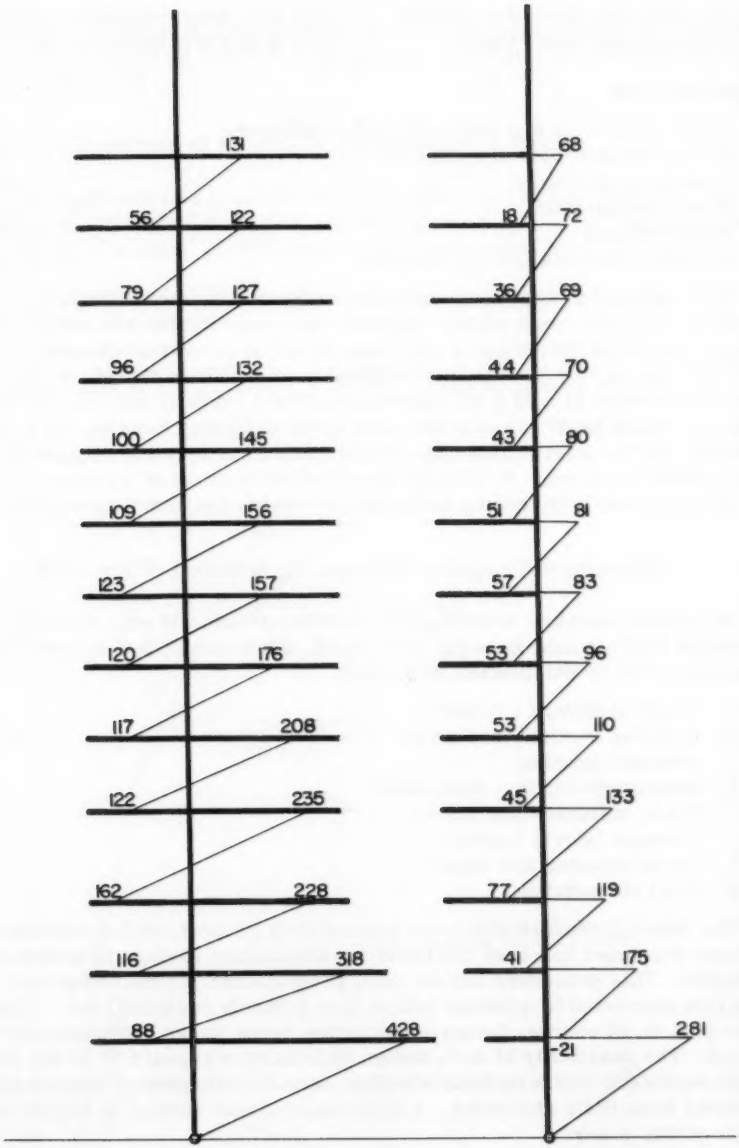
In practice we have found that the results are usually only moderately sensitive to the actual member sizes used. This means that an overall design procedure can be established as follows:

1. Select arbitrary I values
2. Compute vertical load cases
3. Reselect member
4. Recompute vertical load cases
5. Check members and adjust
6. Compute lateral loading
7. Check member and adjust
8. Final computation.

The above procedure yields very accurately proportioned members and in at least two cases has been the basis for substantial savings in structures so designed. This procedure can be itself programmed for the computer, and the program converted to a design rather than a purely analytical tool. The ultimate goal is, of course, design optimization using linear programming techniques. The possibility of such design optimization appears to be the field to which engineers will turn their attention once the drudgery of computation is removed from their shoulders. A discussion of this subject is beyond the scope of this paper.

Discussion of Wind Analysis Results

From a review of the various investigations included herewith it may be seen that there is wide divergence in the results obtained by analytical



FRAME No. 5

COLUMN MOMENTS IN TALL STRUCTURE WITH STIFF COLUMNS HINGED BASES

FIG. No. 11

methods and approximate manual methods in some cases while in others agreement is good.

Out of this study two comments can be made:

1. That the general behaviour of a frame can be very sharply affected, especially in its lower stories by the ratio of column to girder inertias.
2. That the base fixity conditions play a major part in determining the behaviour of a frame.

There are two extreme frame conditions:

1. The flagpole, in which the stiffness of the column with respect to the horizontal members is infinite.
2. The rigid floor system, in which the stiffness of the horizontal members with relationship to the columns is infinite.

Or, expressed in other terms, the case of the infinitely stiff column or the infinitely stiff transverse girder.

Fig. 12 illustrates the fundamental difference in behaviour in these two types of frame.

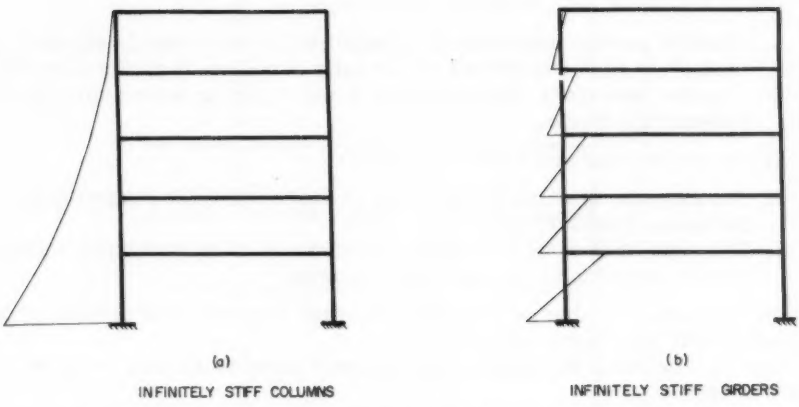
Actual frames will, of course, lie in between these two extremes. In general, heavily loaded short span, low height frames will tend towards the infinitely stiff beam type behaviour. Tall frames with wide bay spacings and relatively lightly loaded floor systems will tend towards the flagpole type of pattern. Also in this category are tall flat slab buildings and other structures in which the transverse system depth is shallow with respect to that of the column system.

An aggravating feature is that the higher the buildings are, the more important and the stiffer the columns in the lower floors become in relationship to their respective transverse beams.

The major area of uncertainty in frame behaviour would appear to be in the lower two or three floors. Above the third or the fourth floor the moments appear to settle into a fairly regular pattern.

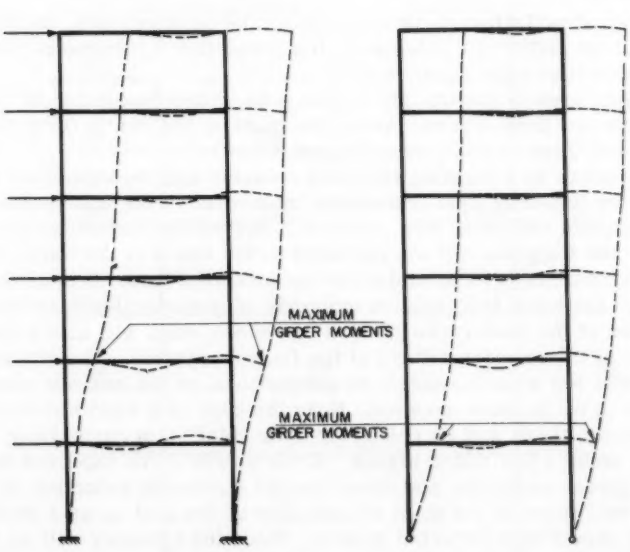
The behaviour of a building with stiff columns may be visualized as that of two flagpoles linked by light transverse members. If the transverse members are of negligible stiffness, then, obviously, the entire moment system will be carried by the flagpoles and the moments at the bases of the flagpoles will be that of two cantilevers each subject to one half total load. It is obvious that this system produces base column moments of considerable magnitude. As the stiffness of the transverse girders increases, each will take a small amount out of the moment pattern of the flagpole system. The amount of moment taken out will, of course, be proportional to the angle of rotation in the column at the point in question. Near the base of a rigid column fixed to its foundations, there will be relatively little rotation in comparison to that which will occur a few floors higher. It can therefore be expected that the maximum girder moments, and therefore the maximum reduction of column moments, will occur at the point of inflection of the system as a whole. Fig. 13 shows a sketch of a deflected system. Since the tendency will be for the girders to try to maintain the columns vertical, the system will tend towards the vertical towards the top in the zone of the lesser loads. This deflection pattern can readily be verified by a simple model test.

The base conditions have a very pronounced effect on the lateral load moments on the first or the second floors. Fig. 13 will show readily the difference that exists in a particular frame when the columns are fixed and



EFFECT ON COLUMN MOMENTS OF COLUMN/GIRDER STIFFNESS WITH FRAME ANALYSED AS A UNIT. LATERAL LOADING.

FIG. No. 12



EFFECT OF FIXED AND HINGED BASES ON LOCATION OF MAXIMUM GIRDER MOMENTS UNDER LATERAL FRAME LOADING

FIG. No. 13

when the columns are not fixed. This situation is, we believe, a good argument in favour of plastic design. It is obvious that structures of this type have performed satisfactorily in the past. It is also obvious that the only reasons that they have done so is that they have, in fact, behaved as plastic structures regardless of whether the designer was himself in favour of or presumed to tolerate the use of plastic design procedures. The application of the approximate elastic design or the portal methods to the case shown in Fig. 2 would result in a structure which would be probably quite safe on an ultimate design basis, but which would be quite unsatisfactory on an elastic design basis. The structures designed elastically by approximate methods could probably only be justified in reality on the basis of plastic design!

The effect of repeated wind loads and stresses on a major structure in which the stresses are obviously going to continue to be in the very high ranges must be considered in relation to possible fatigue conditions. If the wind stresses are very high in relation to vertical load stresses, we must of course check the structure for potential fatigue hazards.

Tall structures in which base fixity exists, whether or not considered as such by the designer could be liable to stresses entering the range of potential fatigue failure even with partial wind loading. There is insufficient information as yet to draw any conclusions, but it would seem to the author that plastic design can only be justified if we are certain that highly recurrent loadings while the structure is still behaving elastically can cause no problems.

Base conditions will, of course, be rightly questioned by many. It would seem quite arbitrary to expect a frame to cease behaving as an integral continuous structural unit at the base plate level. The existence of a complete pin at the foundation would appear to be neither possible nor desirable. Such an absolute hinge would seem to introduce very high moment stresses in certain types of frames in the columns at the first, second and third floor levels.

Some degree of fixity seems desirable to reduce the first, second and third floor moments. Yet the assumption of a perfectly rigid base can give rise to moments of a very high order.

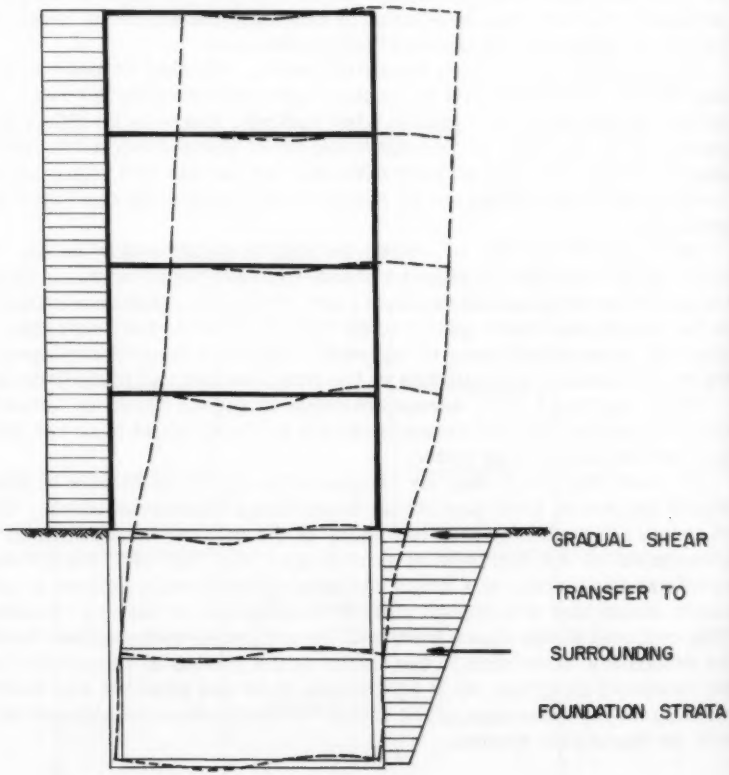
We feel, therefore, that the analysis of a major multistory building frame should include at least part of the foundation or basement levels. This will, of course, of necessity lead to a study of the manner in which shear loads are transferred to the surrounding ground area (see Fig. 14). Except when the structure is on rock, in a major building of 30 or more stories in height, the entire shear load will probably not be transferred at the first foundation level. This removal of the shear loads will be progressive throughout the depth of the foundation in relation to the ability of the ground to absorb the load. With the computer program, as it now stands, it is now possible and desirable to investigate the behaviour of the frame structure when considered as integral with its foundation system.

Validity of Lateral Load Assumptions

The use of computers in analyses causes us to reevaluate the validity and importance of lateral load assumptions. We must beware of the vicious circle that starts with the assumption that lateral loads are only rough approximations at best and concludes, therefore, that an exact analysis is not warranted, and then goes on to state that a more accurate load assumption is not

warranted because the analysis we can make is only approximate at best. It would seem that the logical conclusion of this chain of argument would be the imposition of arbitrary moments at the top and bottom of all columns. This would be far simpler than the academic exercise of piling approximation upon approximation and deluding ourselves on the validity of our design computations. This entire approach has been forced upon use by the past impracticality of making an exact analysis but it should not be considered as a desirable or a permanent state of affairs. While admitting that design office procedures must be practical, we surely must also feel as engineers that only by knowing how a structure behaves and by investigating the nature of loadings can we achieve progress.

We have made a number of investigations, details of which are beyond the scope of this paper, that indicate that local variations in intensity are



COMBINED ACTION UNDER LATERAL LOAD OF SUPERSTRUCTURE AND SUBSTRUCTURE

FIG. No. 14

unimportant provided that the centroid of application of the total wind force is approximately correct. It may be shown that there may be over 100% difference in the intensity between floors in a structure 30 to 40 floors in height without varying base moment conditions by more than 2 or 3% provided that the total force exerted against the building and its location is kept constant. The same reasoning applies to seismic activity. What we really need to know is the intensity and centre of action of the force coupled with a very approximate idea of the distribution.

CONCLUSION

The electronic computer has permitted a practical approach to the exact elastic solution of multistory frames. It has made it possible to analyze the effect of vertical loadings in a systematic fashion, a possibility which should reduce office errors, increase the accuracy of design structures, and permit the filing of complete designs on tape in such a manner as to be readily reviewed in cases of change in either structure or the loading.

Insofar as lateral loadings are concerned, it has placed at the disposal of the designer a procedure which can be used in regular office design. The engineer is no longer obliged to use approximations in such analyses. This lateral load analysis has pointed to several interesting features and characteristics of certain types of structures and has shown that there exist very good possibilities of high stresses in the lower areas of very tall multistory structures. These stresses will apparently be reduced with plastic redistribution and many existing structures when subject to rigorous analysis could probably only be justified on the basis of plastic design. There is reason to believe, however, that fatigue conditions may occur occasionally which could affect the life spans of some tall multistory buildings.

It also must be remembered that the use of a computer in a program such as described in this paper is merely an engineering tool which must always be used at the discretion of the designer, who should never allow the computer to become a substitute for good engineering judgment and common sense. He can, however, make use of the results obtained on such a machine to form a better basis for the formulation of his own judgment and opinion.

BIBLIOGRAPHY

1. "Continuous Frames of Reinforced Concrete", by Hardy Cross and N. D. Morgan, John Wiley and Sons Inc., New York 1932.
2. "Statically Indeterminate Structures", by Jack R. Benjamin; McGraw-Hill, 1959.
3. "Statically Indeterminate Structures", by C. K. Wang; McGraw-Hill, 1953.
4. "Relaxation Methods", by D. N. de G. Allen, McGraw-Hill 1954 (See also extensive bibliography therein).
5. "Rigid Frame Analysis with the Aid of Digital Computers", by E. Czerniak. Proceedings Paper 1634 ASCE May 1958.
6. "A Circuit Analysis of Laterally Loaded Continuous Frames", by Frank Baron, Proceedings Paper 1147, ASCE.

7. "Wind Stresses by Slope Deflection and Converging Approximations", by John E. Goldberg, Paper 1878, Transactions ASCE #99, 1934, page 962.

8. "Modern Mathematics for the Engineer", Chapter 13 and 17. Bechenbach, Editor. McGraw-Hill 1956.

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ASEISMIC DESIGN OF STRUCTURES BY RIGIDITY CRITERION[a]

Closure by Edward Y. W. Tsui

EDWARD Y. W. TSUI,[1] M. ASCE.—The author appreciates the discussion of Mr. Eremin, who pointed out the design coefficients used in the analysis of seismic stresses for bridge structures supported on different foundations in California. It is interesting to note that these coefficients are low, which imply, based on the author's view point, that the rigidity of the structures considered is quite high (or extremely flexible).

Mr. Eremin mentioned that the author assumed the seismic damping force acting along the center of gravity of the single mass structure. This statement is apparently not true. It should be pointed out that the equivelent viscous damping coefficient c as shown in Figs. 2 and 7 is schematic, the actual force considered being $c\dot{z}$, where $\dot{z}$ is the relative velocity of the ground to the mass of structure.

In regard to damping forces in a bridge structure, Mr. Eremin stated that useful conclusions may be derived from observation of the bridge failures during the recent earthquake shocks in California. Since the determination of damping capacity of, or the dissipation of energy in a structure is a difficult task, it would be a great contribution to this new science if this could be derived. Furthermore, bridge failures have been observed, whether or not the main reason for this was due to the negligence of the rigidity of structure in design is yet a question.

Mr. Eremin's last question as to determine which features of an industrial or specialized type of structures are to be included in the analysis for seismic stresses may be answered, as follows:

1. Determination of the rigidity or the fundamental period of the structure.
2. Evaluate a reasonable value for η —the fraction of critical damping of the structure. (Prof. L. S. Jacobsen of Stanford University recently published the "Frictional Effects in Composite Structures subjected to Earthquake Vibrations," which is one of the latest references on damping in structures.)

   Once the above features are determined, the seismic shear of the structure can be interpolated from Fig. 7. It is suggested to use 1/2, 1/4, and 1/8 of the values shown in this figure for Zone III, II and I respectively as specified by the Uniform Building Code.

Finally, the author might mention that the above method of analysis was applied to determine the seismic response of a gas-cooled reactor structure

a. Proc. Paper 1942, February, 1959, by Edward Y. W. Tsui.

1. Head, Applied Mechanics-Dynamics, Structures Dept., Lockheed Missiles and Space Div., Palo Alto, Calif.

for a nuclear power plant to be located in the active earthquake zone of this country.

CORRECTION: Pages 93 and 95, $Z(y) = Z_1 \cos \frac{Y}{2L}$ should read $(Z(y) = Z_1 \cos \frac{\pi y}{2L}$.

THE PLASTIC METHOD OF DESIGNING STEEL STRUCTURES[a]

Discussion by H. L. Su

H. L. SU.[1]—It has been pointed out recently[1,2] that the so called collapse method of structural design is based upon an illogical foundation which is indeed on the brink of collapse. However, in the synopsis, it is stated that the abortive attempt of the Steel Research Committee in 1936 to produce an acceptable rational elastic method of design led "logically" to the consideration of the behavior of structures in the plastic range. How can the shortcomings of a method be used to justify another, which is even farther from reality?

Now let us examine the arguments advanced by the "plasticians."

1. The author mentioned the bitter experience of the Steel Structures Research Committee that most practical structures are highly redundant and their elastic behavior is so complex that elastic theory cannot lead to anything like a satisfactory direct design method. Nevertheless, it should be borne in mind that this Committee could not, unfortunately, benefit from the modern computers, either digital or analog. On the other hand, the collapse mechanism of such highly redundant structures will be more complex than their elastic behavior. Suppose that the required number of plastic hinges is 50 and that there can only be 100 possible hinge-locations. Simple arithmetic will reveal that there will be $1.015 \times 10 = 100!/50!50!$ different modes of collapse, and billions of designers will be required to spend their whole life on analyzing such a structure. However, 49 unknowns present little difficulty in the elastic method, particularly with the assistance of modern computers.

It is interesting to note that such a plastic design is even very difficult, if not impossible at present, to program either for a digital or for an analog computer.

2. The author seemed to have identified the elastic method of design with the pin-joint assumption, in spite of the fact that designers of the present generation seldom hear of a "pin-joint design" of a rigid-joint structure. It was stated in the paper that the orthodox method of elastic design takes no cognizance of the possibility of relative sinking of supports, of residual or locked-up stresses or, in fact, of several other inconvenient factors including the inevitable variations in practical connections.

The elastic method of design has been occasionally abused in the past. Many important aspects in the design problem have indeed been ignored. Nevertheless, such abuses and neglects are by no means incurable. On the other hand, there exist a great many "inconvenient factors" in the plastic method.

a. Proc. Paper 2005, April, 1959, by J. F. Baker.

1. Analogue Laboratory, Civ. Eng. Dept., Imperial College, London, England.

It is completely true that when a structure collapses, the sinking of supports, and indeed many other things become immaterial. The point is, however, whether such effects can be written off merely by use of a collapse diagram on the drawing board. If not, the designer relys on an arbitrary artifice to disguise the physical reality and to satisfy himself with some false sense of security. The "elastician" is, at least, honest in neglecting such effects, which he considered too tedious to be included in the design.

As to the joint rigidity, the author's statement is unfair. Is there any ground to immunize the plastic method from the imperfection of connections if it fails the elastic method? In fact, such imperfection can easily be taken into account in the elastic design by modifying the joint stiffness in calculation; whereas in the case of the plastic method, such an improvement will wash away the lustre of simplicity claimed that is.

3. Various experiments on the collapse mechanism have been reported in recent years, and nobody will doubt the validity of these test results under their specified conditions. If a designer only needs to aim at such tests, even from the miniature to the full scale, the plastic method advocated by the author is indeed a logical proposition. Unfortunately, the designer has to face reality, which cannot be simply simulated by these tests. It is far from the truth that "the behaviour at collapse of a structure can be foretold with great accuracy and simplicity." The reason is simply that except in a few types of structures, the actual loading upon a structure always remains an uncertainty. As the collapse mechanism is a function of the applied loading, how can the fate of the structure "be foretold with great accuracy and simplicity"?

4. The author reported many practical examples of the plastic design, and found that "deflexions and other conditions under working loads are satisfactory." Is this a clear enough indication that such structures still behave either elastically or pseudo-elastically, but not plastically? So the structure designed to collapse is made not to collapse after all. If this philosophy is acceptable, who should we not develop a viscous method of design? It is possible to assume a structure to be a Newtonian, or even better a Maxwellian, liquid on the drawing board, and then to use a load factor, say 3, or 4, to prevent the actual structure from flowing. This kind of viscous flow is no secret at all, and there will be little difficulty in extracting experimental evidence to support such a viscous method of design.

The core of the question is whether a design should be based on its expected performance or on some imaginary behavior to be avoided. If a structure is expected not to collapse during its service life. should the designer only consider a case which should almost never occur?

5. The plastic method of analysis can, however, be employed as a method of checking, although such checking is seldom necessary. It should then be noted that the composite action of the structure will invariably alter the collapse mechanism of the main framework. When a building framework is "bare," the loading will not reach its maximum. When the framework is clad, the composite action comes into play. Therefore, the plastic method of analysis may even be unrealistic in checking a design of clad frameworks, such as the multi-story building in Cambridge University described by the author.

The author mentioned that "the amount of energy which can be absorbed by

a continuous ductile structure subjected to plastic bending is very large." This is true only if plastic bending is acceptable. It is not incidental that this collapse design was made on air raid shelters, which will undoubtedly fall into a different category from the ordinary structures so far as the probability of safety is concerned.

6. It should be pointed out that the plastic method of design is a heterodox development of the working-stress concept. The pioneers in the engineering profession were correct in introducing a factor of safety to allow for the uncertain influence possibly appearing during the service life of a design. Their successors have so far been correct also in referring to this factor as the factor of ignorance, which is to incorporate the variation of the quality of material and workmanship, the inaccuracy of the physical theory, the uncertainty of future disturbance, etc. Unfortunately, after such a correct diagnosis, no cure was thought of by the engineers.

The stochastic nature of many of the above uncertain factors is only qualitatively appreciated. This is a consequence of the facts that the engineers lack the knowledge of statistics, and that the statistics itself has not yet been sufficiently developed to provide a ready answer. Recently, some successful attempts have been made in formulating a method to quantitatively interpret such uncertainty.(3,4,5,6,2) This approach will no doubt prove useful, practicable and economical particularly when the statistical data concerned become available.

Many "plasticians" have claimed that the load factor, a form of the safety factor, obtained in a plastic design, is a "true" one. They have forgot the probabilistic meaning of this factor, the true value of which can never be found. Even if it is found by chance, it can never be identified. Theoretically speaking, only a most probable factor or a factor corresponding to a certain probability can be obtained. As a matter of fact, all of their elaborate maneuvers are submerged in this "ture" factor. The structure designed in such a way will stand still, and cannot be mobilized in accordance with the dictates of the "plasticians."

It is possible, of course, to design by means of the statistical approach any object which is to be on the verge of collapse or which actually collapses. There are numerous examples of this kind in the manufacturing industry, such as metal cutting, wire drawing, etc., in which the yield mechanism is indispensible. However, in the building industry, the situation is different. The author has now appreciated that "the occupier of a building is, of course, more interested in its behaviour under working loads than at collapse." Then why should we bother to adopt the conditions at collapse as the design criter criterion?

REFERENCES

1. Su, H. L., On Plastic Method, Proc. Inst. Civ. Eng., London, Jul. 1959, p. 410.
2. Su, H. L., "Design by Quantitative Safety Factor," Proc. A. S. Mech. E. Paper No. 59-A-48.
3. Su, H. L., "Statistical Approach to Structural Design," Proc. Inst. Civ. Eng., London, Jul. 1959, p. 353.

4. Pugsley, A. G., "Concepts of safety in structural engineering," J. In. Civ. Eng., v. 36, p. 5, Mar. 1951.

5. Pugsley, A. G., "Structural Safety," J. Roy. Aero. Soc., v. 59, 1955.

6. Freudenthal, A. M., "The Safety of Structures," Tran. ASCE, v 112, p. 1337, 1956.

ANALYSIS OF TWO-COLUMN SYMMETRICAL BENTS[a]

Discussion by Charles O. Heller

CHARLES O. HELLER,[1] A.M. ASCE.—The author should be complimented for bringing the subject of analysis of one-day, multi-story, rectangular towers to the attention of the profession. However, his presentation appears to be neither original nor complete.

There is a long line of investigators who paid considerable attention to the special nature of the problem of an unsymmetrically loaded frame of the above type, and of those who took advantage of the symmetry and antisymmetry of the loaded frame. The idea of resolving a symmetrical, one-bay, multi-story frame, unsymmetrically loaded, into a symmetrical and an antisymmetrical system was shown by Andrée,(1) Newell,(2) Bayer,(3) Naylor,(4) Pei,(5) and others. The modified method of moment distribution for analyzing such a frame was originally introduced by Perri,(6) Hadley,(7) and Kavanagh.(8) Similar material was later introduced under a new name by Grinter and Tsao.(9) The application of this approach was then fully explained by Parcel and Moorman(10) through a numerical example which is essentially identical to that of the author. Kupferschmid(11) also presented the author's method and extended it to the analysis of multi-story towers with gabled roofs and Vierendeel trusses.

Recently, Goldberg(12) suggested an analysis of one-bay, multi-story, rectangular frames by means of three-slope equations. Modifications of Goldberg's approach and some additional possibilities were demonstrated by Nubar,(13) Sobotka,(14) Chang.(15) The author(16) also contributed a discussion to Goldberg's publication, and his paper, "Analysis of Two-Column Symmetrical Bents," was a direct outgrowth of this discussion. Yet, no credit was given to Goldberg for having stimulated this study.

The writer wished to underline the fact that, in reviewing the background of this problem, his sole intention is to make the author's paper more complete and to provide the students of this subject with a more extensive list of references.

In the following discussion, the writer introduces a general derivation of this modified method of moment distribution for one-bay, multi-story, rectangular frames:

1. Definition of the Problem

A multi-story two-column symmetrical bent, acted upon by a general system of loads, is considered (Fig. 1-1). The cross-sections of both columns and girders are constant in a given span. Since the structure is symmetrical

a. Proc. Paper 2037, May 1959, by B. R. Cooke.

1. Graduate Assistant, Oklahoma State Univ., Stillwater, Okla.

and unsymmetrically loaded, the resolution of the system into a symmetrical (Case I) and antisymmetrical (Case II) condition offers many advantages, as has been shown by Andrée,(1) Newell,(2) and Beyer,(3) Naylor(4) and Pei.(5)

2. Symmetrical Condition-Case I

The deformation curve is symmetrical with respect to the vertical axis of the frame. Thus, the joint rotations of the left side are symmetrical with their counterparts of the right side and no translation takes place. The end moment equations of a typical portion of this frame (Fig. 2-2) are:

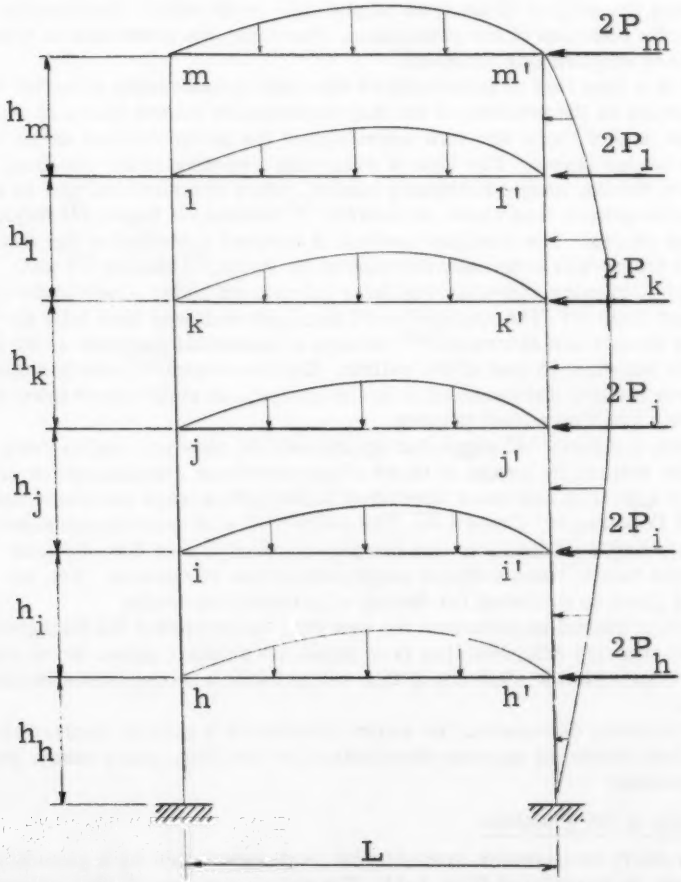


Fig. 1-1

$$
\left.
\begin{aligned}
M_{kj}^{(I)} &= 4Ek_k\,\theta'_k + 2Ek_k\,\theta'_j + FM_{kj}^{(I)} \\
M_{jk}^{(I)} &= 4Ek_k\,\theta'_j + 2Ek_k\,\theta'_k + FM_{jk}^{(I)} \\
M_{jj}^{(I)} &= 2Ek_{jj}\,\theta'_j + FM_{jj}^{(I)} \\
M_{ji}^{(I)} &= 4Ek_j\,\theta'_j + 2Ek_j\,\theta'_i + FM_{ji}^{(I)} \\
M_{ij}^{(I)} &= 4Ek_j\,\theta'_i + 2Ek_j\,\theta'_j + FM_{ij}^{(I)} ,
\end{aligned}
\right\} \qquad (2\text{-}1)
$$

in which

$$k_k = \frac{I_k}{h_k} \qquad k_{jj} = \frac{I_{jj}}{L} \qquad k_j = \frac{I_j}{h_j}$$

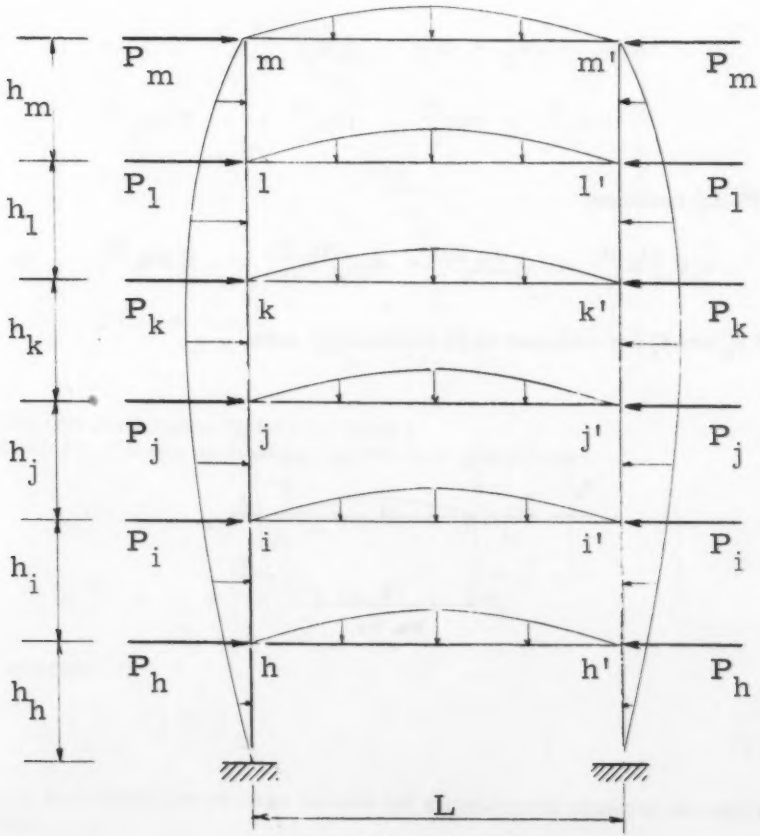


Fig. 2-1

$FM^{(I)}$ = Fixed end moment due to loads (symmetrical case). The other symbols are those commonly used in the slope deflection method.

From the equilibrium of joint j (any joint):

$$M_{jk}^{(I)} + M_{jj}^{(I)} + M_{ji}^{(I)} = 0 \qquad (2\text{-}2a)$$

or in terms of Eqs. (2-1),

$$2Ek_k\theta'_k + E(4k_k + 2k_{jj} + 4k_j)\theta'_j + 2Ek_j\theta'_i = \\ = - FM_{jk}^{(I)} - FM_{jj}^{(I)} - FM_{ji}^{(I)} . \qquad (2\text{-}2b)$$

With notations

$$C_{kj}^{(I)}K_{kj}^{(I)} = 2Ek_k \quad \Big| \quad C_{ij}^{(I)}K_{ij}^{(I)} = 2Ek_j$$

$$E(4k_k + 2k_{jj} + 4k_j) = \Sigma K_j^{(I)}$$

$$- FM_{jk}^{(I)} - FM_{jj}^{(I)} - FM_{ji}^{(I)} = - \Sigma FM_j^{(I)} ,$$

Eq. (2-2b) becomes:

$$\theta'_k C_{kj}^{(I)}K_{kj}^{(I)} + \theta'_j \Sigma K_j^{(I)} + \theta'_i C_{ij}^{(I)}K_{ij}^{(I)} = - \Sigma FM_j^{(I)}. \qquad (2\text{-}2)$$

If θ'_k and θ'_i are assumed to be temporarily zero,

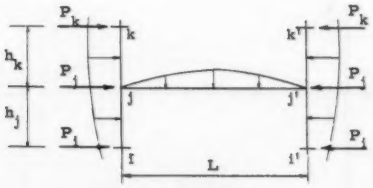


Fig. 2-2

$$\theta'_j = - \frac{\Sigma FM_j^{(I)}}{\Sigma K_j^{(I)}} \quad . \tag{2-3}$$

Then, the end moments at j (Eqs. 2-1) reduce to

$$\left.\begin{aligned} M_{jk}^{(I)} &= - D_{jk}^{(I)} \Sigma FM_j^{(I)} + FM_{jk}^{(I)} \\ M_{jj}^{(I)} &= - D_{jj}^{(I)} \Sigma FM_j^{(I)} + FM_{jj}^{(I)} \\ M_{ji}^{(I)} &= - D_{ji}^{(I)} \Sigma FM_j^{(I)} + FM_{ji}^{(I)} \quad . \end{aligned}\right\} \tag{2-4}$$

The symbols

$$\left.\begin{aligned} D_{jk}^{(I)} &= \frac{K_{jk}^{(I)}}{\Sigma K_j^{(I)}} = \frac{k_k}{k_k + \frac{1}{2} k_{jj} + k_j} \\ D_{jj}^{(I)} &= \frac{K_{jj}^{(I)}}{\Sigma K_j^{(I)}} = \frac{\frac{1}{2} k_{jj}}{k_k + \frac{1}{2} k_{jj} + k_j} \\ D_{ji}^{(I)} &= \frac{K_{ji}^{(I)}}{\Sigma K_j^{(I)}} = \frac{k_j}{k_k + \frac{1}{2} k_{jj} + k_j} \end{aligned}\right\} \tag{2-5}$$

are the new distribution factors for joint j.

Similarly, the end moments at the far ends k and i are:

$$\left.\begin{aligned} M_{kj}^{(I)} &= - C_{jk}^{(I)} D_{jk}^{(I)} \Sigma FM_j^{(I)} + FM_{kj}^{(I)} \\ M_{ij}^{(I)} &= - C_{ji}^{(I)} D_{ji}^{(I)} \Sigma FM_j^{(I)} + FM_{ij}^{(I)} \quad . \end{aligned}\right\} \tag{2-6}$$

The symbols

$$C_{jk}^{(I)} = \frac{1}{2} \qquad C_{ji}^{(I)} = \frac{1}{2} \tag{2-7}$$

are the well-known carry-over factors for a member of constant moment of inertia.

Since θ'_i, is eliminated due to the symmetry of deformation, no carry-over occurs in the jj' direction.

The formulas (2-4) and (2-6) are perfectly general and can be applied to any joint of Fig. 2-1. Using the constants and fixed end moments from these equations, the moment distribution can be readily applied.

3. Antisymmetrical Condition-Case II

The deformation curve is antisymmetrical with respect to the vertical axis of the frame. Therefore, the joint rotations of the left side are antisymmetrical with their counterparts on the right side and each story translates an amount of Δ in the direction of the loads. The end moment equations of a typical portion of this frame are:

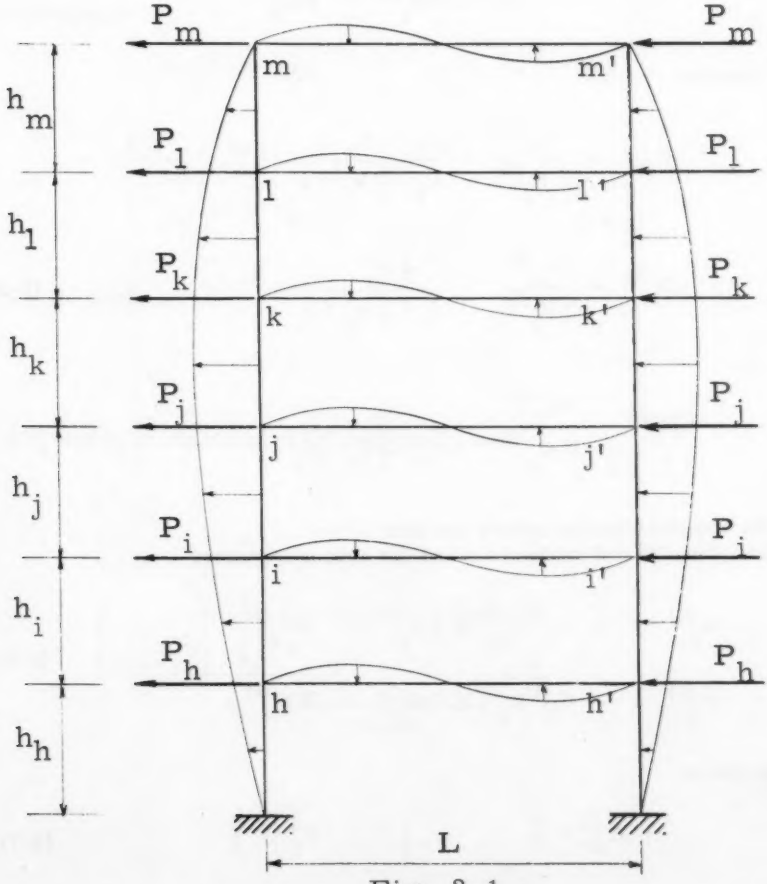


Fig. 3-1

$$\left.\begin{aligned}
M_{kj}^{(II)} &= 4Ek_k\theta_k'' + 2Ek_k\theta_j'' + 6Ek_k\psi_k + FM_{kj}^{(II)} \\
M_{jk}^{(II)} &= 4Ek_k\theta_j'' + 2Ek_k\theta_k'' + 6Ek_k\psi_k + FM_{jk}^{(II)} \\
M_{jj}^{(II)} &= 6Ek_{jj}\theta_j'' + FM_{jj}^{(II)} \\
M_{ji}^{(II)} &= 4Ek_j\theta_j'' + 2Ek_j\theta_i'' + 6Ek_j\psi_j + FM_{ji}^{(II)} \\
M_{ij}^{(II)} &= 4Ek_j\theta_i'' + 2Ek_j\theta_j'' + 6Ek_j\psi_j + FM_{ij}^{(II)}.
\end{aligned}\right\} \quad (3\text{-}1)$$

where

$$\psi_k = \frac{\Delta_k}{h_k} \qquad \psi_j = \frac{\Delta_j}{h_j}$$

with Δ_k = relative translation of the joint k

Δ_j = relative translation of the joint j

and the other symbols similar to those in Eqs. (2-1).

From the equilibrium of horizontal forces at jj'

$$\frac{12Ek_k}{h_k}(\theta_j'' + \theta_k'' + 2\psi_k) + 2FV_{jk}^{(II)} - \Sigma V_j^{(II)} = 0. \quad (3\text{-}2a)$$

where

$FV_{jk}^{(II)}$ = horizontal shear of fixed-end member $\overline{jk}$ at j due to loads,

and

$\Sigma V_j^{(II)}$ = sum of all horizontal loads above jj'.

From Eq. (3-2a)

$$6Ek_k\psi_k = -3Ek_k(\theta_j'' + \theta_k'') - \frac{h_k FV_{jk}^{(II)}}{2} + \frac{h_k \Sigma V_j^{(II)}}{4}. \quad (3\text{-}2)$$

Similarly,

$$6Ek_j\psi_j = -3Ek_j(\theta_i'' + \theta_j'') - \frac{h_j FV_{ij}^{(II)}}{2} + \frac{h_j \Sigma V_i^{(II)}}{4}. \quad (3\text{-}3)$$

In terms of Eqs. (3-2, 3), the end moments at j (Eqs. (3-1) become:

$$\left.\begin{aligned} M_{jk}^{(II)} &= Ek_k\theta''_j - Ek_k\theta''_k + GM_{jk}^{(II)} \\ M_{jj}^{(II)} &= 6Ek_{jj}\theta''_j + FM_{jj}^{(II)} \\ M_{ji}^{(II)} &= Ek_j\theta''_j - Ek_j\theta''_i + GM_{ji}^{(II)} \end{aligned}\right\} \quad (3\text{-}4)$$

where

$GM_{jk}^{(II)}$ = fixed end moment of the guided member $\overline{jk}$ at j (Fig. 3-2),

$GM_{ji}^{(II)}$ = fixed end moment of the guided member $\overline{ij}$ at j (Fig. 3-3).

The general formula for

$$GM_{jk}^{(II)} = FM_{jk}^{(II)} - \frac{h_k FV_{jk}^{(II)}}{2} + \frac{h_k \Sigma V_j^{(II)}}{4} \quad (3\text{-}5)$$

and for

$$GM_{ji}^{(II)} = FM_{ji}^{(II)} - \frac{h_j FV_{ij}^{(II)}}{2} + \frac{h_j \Sigma V_i^{(II)}}{4} . \quad (3\text{-}6)$$

From the equilibrium of moments at joint j (any joint):

$$\left.\begin{aligned} &- Ek_k\theta''_k + E(k_k + 6k_{jj} + k_j)\theta''_j - Ek_j\theta''_i = \\ &= - GM_{jk}^{(II)} - FM_{jj}^{(II)} - GM_{ji}^{(II)} . \end{aligned}\right\} \quad (3\text{-}7)$$

Introducing slope deflection notations for Eq. (3-7):

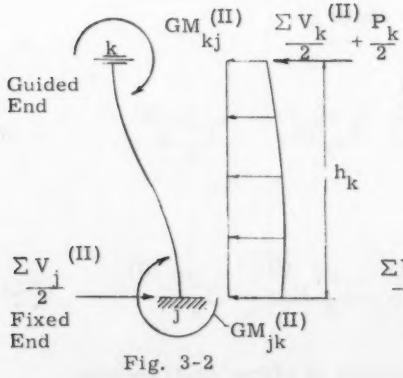


Fig. 3-2

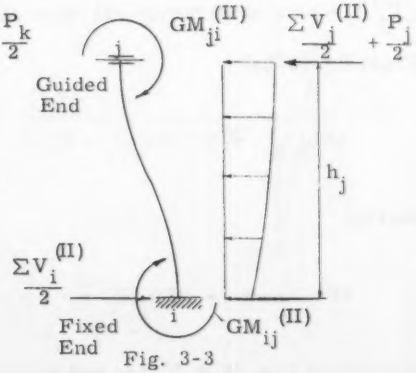


Fig. 3-3

$$C_{jk}^{(II)}K_{jk}^{(II)} = -Ek_k \quad \Big| \quad C_{ji}^{(II)}K_{ji}^{(II)} = -Ek_j$$

$$E\,(k_k + 6k_{jj} + k_j) = \Sigma K_j^{(II)}$$

$$GM_{jk}^{(II)} + FM_{jj}^{(II)} + GM_{ji}^{(II)} = \Sigma FM_j^{(II)}$$

reduces that equation to:

$$\theta_k'' C_{kj}^{(II)}K_{kj}^{(II)} + \theta_j'' \Sigma K_j^{(II)} + \theta_i'' C_{ij}^{(II)}K_{ij}^{(II)} = -\Sigma FM_j^{(II)}. \qquad (3\text{-}8)$$

If θ_k'' and θ_i'' are assumed to be temporarily zero,

$$\theta_j'' = -\frac{\Sigma FM_j^{(II)}}{\Sigma K_j^{(II)}}, \qquad (3\text{-}9)$$

then the end moments at j (Eqs. 3-4) reduce to

$$\left.\begin{aligned} M_{jk}^{(II)} &= -D_{jk}^{(II)}\Sigma FM_j^{(II)} + GM_{jk}^{(II)} \\ M_{jj}^{(II)} &= -D_{jj}^{(II)}\Sigma FM_j^{(II)} + FM_{jj}^{(II)} \\ M_{ji}^{(II)} &= -D_{ji}^{(II)}\Sigma FM_j^{(II)} + GM_{ji}^{(II)} \end{aligned}\right\} \qquad (3\text{-}10)$$

The symbols

$$\left.\begin{aligned} D_{jk}^{(II)} &= \frac{K_{jk}^{(II)}}{\Sigma K_j^{(II)}} = \frac{k_k}{k_k + 6k_{jj} + k_j} \\ D_{jj}^{(II)} &= \frac{K_{jj}^{(II)}}{\Sigma K_j^{(II)}} = \frac{6k_{jj}}{k_k + 6k_{jj} + k_j} \\ D_{ji}^{(II)} &= \frac{K_{ji}^{(II)}}{\Sigma K_j^{(II)}} = \frac{k_j}{k_k + 6k_{jj} + k_j} \end{aligned}\right\} \qquad (3\text{-}11)$$

are the new distribution factors for joint j.

Similarly, the end moments at the far ends k and i are:

$$M_{kj}^{(II)} = C_{jk}^{(II)} D_{jk}^{(II)} \Sigma FM_j^{(II)} + GM_{kj}^{(II)}$$

$$M_{ij}^{(II)} = C_{ji}^{(II)} D_{ji}^{(II)} \Sigma FM_j^{(II)} + GM_{ij}^{(II)} . \qquad (3\text{-}12)$$

The symbols

$$C_{jk}^{(II)} = -1 \quad \Big| \quad C_{ji}^{(II)} = -1 \qquad (3\text{-}13)$$

are the new carry-over factors for a member of constant moment of inertia.

The equations (3-10) and (3-12) are perfectly general and can be applied to any joint of Fig. (3-1). Using the constants and fixed end moments of these equations, and evaluating the guided moments[17] properly, the moment distribution can be readily applied. The guided moments for the most common loading conditions are shown in Table (3-1).

The writer wished to express his gratitude to Professor Jan J. Tuma who introduced him to the analysis of one-bay, multi-story, rectangular frames. A method similar to that in this discussion was taught by Professor Tuma at Oklahoma State University.

The writer also wishes to thank Mr. James W. Gillespie, Instructor in the School of Civil Engineering at Oklahoma State University, for his kind advice and aid in proofreading the manuscript of this discussion.

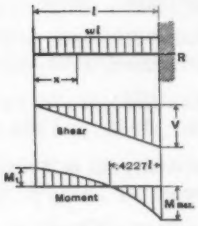
Upon having read this discussion, Professor Tuma brought to the writer's attention a book concerning moment distribution by Kazda,[18] where the solution suggested by the author is mentioned and illustrated by several examples.

REFERENCES

1. Andrée, W. L., Das B = U Verfahren, R. Oldenburg, Munich, 1919.
2. Newell, J. S., "Symmetric and Anti-Symmetric Loadings," Civ. Eng., April, 1939, pp. 249-251.
3. Beyer, K., Die Statik im Stahlbetonbau, Springer, Berlin, 1948, 2nd. Ed., pp. 457-480.
4. Naylor, N., "Sidesway in Symmetrical Building Frames," The Structural Engineer, April 1950, p. 99.
5. Discussion by Pei, Ming-Lung, to "Joint Translation by Cantilever Moment Distribution," Transactions, ASCE, vol. 119, 1954, p. 1208.
6. Perri, Joseph G., "Modified Method of Moment Distribution for Analyzing Compression Members as Parts of Rigid Frames," Ph. D. Thesis, New York University, June 1948.
7. Hadley, Richard H., "Sidesway Analysis of Rectangular Rigid Frames by Component Moment Methods," M. S. Thesis, University of Washington, 1949.
8. Kavanagh, T. C., "Analysis and Design of Columns in Frames Subject to Translation," Column Research Council Report, 1950.

TABLE 3-1
GUIDED BEAMS*

20. BEAM FIXED AT ONE END, FREE BUT GUIDED AT OTHER—UNIFORMLY DISTRIBUTED LOAD



The deflection at the guided end is assumed to be in a vertical plane.

Equivalent Tabular Load . . . . $= \frac{5}{8} wl$

R = V . . . . . $= wl$

V_x . . . . . $= wx$

M max. (at fixed end) . . . . . $= \frac{wl^2}{3}$

M_1 (at guided end) . . . . $= \frac{wl^2}{6}$

M_x . . . . . $= \frac{w}{6}(l^2 - 3x^2)$

Δmax. (at guided end) . . . . $= \frac{wl^4}{24EI}$

Δ_x . . . . . $= \frac{w(l^2 - x^2)^2}{24EI}$

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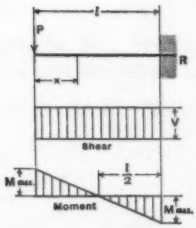
23. BEAM FIXED AT ONE END, FREE BUT GUIDED AT OTHER—CONCENTRATED LOAD AT GUIDED END



The deflection at the guided end is assumed to be in a vertical plane.

Equivalent Tabular Load . . . . = 4P

R = V . . . . . = P

M max. (at both ends) . . . . $= \frac{Pl}{2}$

M_x . . . . . $= P\left(\frac{l}{2} - x\right)$

Δmax. (at guided end) . . . . $= \frac{Pl^3}{12EI}$

Δ_x . . . . . $= \frac{P(l-x)^2}{12EI}(l + 2x)$

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* Steel Construction Manual of the American Institute of Steel Construction, New York, 1953, pp. 372, 373.

9. Grinter, L. E. and Tsao, C. H., "Joint Translation by Cantilever Moment Distribution," Transactions, ASCE, vol. 119, 1954, p. 1195.

10. Parcel, J. I. and Moorman, R. B. B., Analysis of Statically Indeterminate Structures, John Wiley and Sons, Inc., New York, N. Y., 1955, p. 263.

11. Kupferschmid, Victor, Ebene und Räumliche Rahmentragwerke, Springer-Verlag, Vienna, 1952, p. 177.

12. Goldberg, John E., "Lateral Load Analysis of Two-Column Bents," Proceedings, ASCE, vol. 84, No. ST3, Paper 1638, May, 1958.

13. Discussion by Nubar, Yves, to "Lateral Load Analysis of Two-Column Bents," Proceedings, ASCE, vol. 84, ST7, November, 1958, p. 1857-43.

14. Discussion by Sobotka, Zdeněk, to "Lateral Load Analysis of Two-Column Bents," Proceedings, ASCE, vol. 84, ST7, November, 1958, p. 1857-48.

15. Discussion by Chang, I. Chen, to "Lateral Load Analysis of Two-Column Bents," Proceedings, ASCE, vol. 84, November, 1958, p. 1857-56.

16. Discussion by Cooke, B. R., to "Lateral Load Analysis of Two-Column Bents," Proceedings, ASCE, vol. 84, ST8, December 1958, p. 1882-13.

17. Steel Construction Manual of the American Institute of Steel Construction, New York, 1953, pp. 175-201, 372, 373.

18. Kazda, Jaromír, Rozvod Momentů, S.N.T.L., Prague, 1959, pp. 37, 318-327, 363-374.

MATRIX SOLUTION OF BEAMS WITH VARIABLE MOMENTS OF INERTIA[a]

Corrections

CORRECTIONS—Through an oversight, the following list of references for the above-mentioned paper was omitted from the October, 1959, Journal of the Structural Division:

1. Southwell, R. V., "Relaxation Methods in Theoretical Physics" Oxford Press, pp. 16.
2. Salvadori and Baron, "Numerical Methods in Engineering" Prentice Hall Co., pp. 47.
3. Booth, A. D., "Numerical Methods," Butterworths Scientific Publications, Inc.
4. Frazer, Duncan and Collar, "Elementary Matrices," Cambridge University Press.
5. Shedd, T. C. and Vawter, J., "Theory of Simple Structures," John Wiley Co., Chap. VIII.
6. Bisplinghoff, Ashley and Halfman, "Aeroelasticity," Addison-Wesley Co., Chap. II and Appendix A.
7. Maugh, L. C., "Statically Indeterminate Structures," John Wiley Co., pp. 135.
8. Newmark, N. M., "Numerical Procedure for Computing Deflections Moments and Buckling Loads," Trans. ASCE Vol. 108 pp. 1161-1234, 1943.
9. Allen, D. N. de G., "Relaxation Methods," McGraw Hill Co., pp. 39-48.

a. Proc. Paper 2218, October, 1959, by Ming L. Pei.

LATERALLY DEFLECTED COLUMNS[a]

Corrections

CORRECTIONS.--On page 50, in the first line of text following Eq. 15, the term $(1 + u' \sec u')$ should read $(1 + \frac{u'^2}{3} \sec u')$.

On page 59, the equations designated as 42 and 43 should be interchanged.

a. Proc. Paper 2221, October, 1959, by John Sherman.

PROCEEDINGS PAPERS

The technical papers published in the past year are identified by number below. Technical-division sponsorship is indicated by an abbreviation at the end of each Paper Number, the symbols referring to: Air Transport (AT), City Planning (CP), Construction (CO), Engineering Mechanics (EM), Highway (HW), Hydraulics (HY), Irrigation and Drainage (IR), Pipeline (PL), Power (PO), Sanitary Engineering (SA), Soil Mechanics and Foundations (SM), Structural (ST), Surveying and Mapping (SU), and Waterways and Harbors (WW), divisions. Papers sponsored by the Department of Conditions of Practice are identified by the symbols (PP). For titles and order coupons, refer to the appropriate issue of "Civil Engineering." Beginning with Volume 82 (January 1956) papers were published in Journals of the various Technical Divisions. To locate papers in the Journals, the symbols after the paper number are followed by a numeral designating the issue of a particular Journal in which the paper appeared. For example, Paper 1859 is identified as 1859 (HY7) which indicates that the paper is contained in the seventh issue of the Journal of the Hydraulics Division during 1958.

VOLUME 84 (1958)

DECEMBER: 1859(HY7), 1860(IR4), 1861(IR4), 1862(IR4), 1863(SM5), 1864(SM5), 1865(ST8), 1866(ST8), 1867 (ST8), 1868(PP1), 1869(PP1), 1870(PP1), 1871(PP1), 1872(PP1), 1873(WW5), 1874(WW5), 1875(WW5), 1876 (WW5), 1877(CP2), 1878(ST8), 1879(ST8), 1880(HY7)[c], 1881(SM5)[c], 1882(ST8)[c], 1883(PP1)[c], 1884(WW5)[c], 1885(CP2)[c], 1886(PO6), 1887(PO6), 1888(PO6), 1889(PO6), 1890(HY7), 1891(PP1).

VOLUME 85 (1959)

JANUARY: 1892(AT1), 1893(AT1), 1894(EM1), 1895(EM1), 1896(EM1), 1897(EM1), 1898(EM1), 1899(HW1), 1900(HW1), 1901(HY1), 1902(HY1), 1903(HY1), 1904(HY1), 1905(PL1), 1906(PL1), 1907(PL1), 1908(PL1), 1909(ST1), 1910(ST1), 1911(ST1), 1912(ST1), 1913(ST1), 1914(ST1), 1915(ST1), 1916(AT1)[c], 1917(EM1)[c], 1918(HW1)[c], 1919(HY1)[c], 1920(PL1)[c], 1921(SA1)[c], 1922(ST1)[c], 1923(EM1), 1924(HW1), 1925(HW1), 1926 (PL1), 1927(HW), 1928(HW1), 1929(SA1), 1930(SA1), 1931(SA1), 1932(SA1).

FEBRUARY: 1933(HY2), 1934(HY2), 1935(HY2), 1936(SM1), 1937(SM1), 1938(ST2), 1939(ST2), 1940(ST2), 1941(ST2), 1942(ST2), 1943(ST2), 1944(ST2), 1945(HY2), 1946(PO1), 1947(PO1), 1948(PO1), 1949(PO1), 1950(HY2)[c], 1951(SM1)[c], 1952(ST2)[c], 1953(PO1)[c], 1954(CO1), 1955(CO1),1956(CO1),1957(CO1), 1958(CO1), 1959(CO1).

MARCH: 1960(HY3), 1961(HY3), 1962(HY3), 1963(IR1), 1964(IR1), 1965(IR1), 1966(IR1), 1967(SA2), 1968(SA2), 1969(ST3), 1970(ST3), 1971(ST3), 1972(ST3), 1973(ST3), 1974(ST3), 1975(ST3), 1976(WW1), 1977(WW1), 1978(WW1), 1979(WW1), 1980(WW1), 1981(WW1), 1982(WW1), 1983(WW1), 1984(SA2), 1985(SA2)[c], 1986 (IR1)[c], 1987(WW1)[c], 1988(ST3)[c], 1989(HY3)[c].

APRIL: 1990(EM2), 1991(EM2), 1992(EM2), 1993(HW2), 1994(HY4), 1995(HY4), 1996(HY4), 1997(HY4), 1998 (SM2), 1999(SM2), 2000(SM2), 2001(SM2), 2002(ST4), 2003(ST4), 2004(ST4), 2005(ST4), 2006(PO2), 2007 (HW2)[c], 2008(EM2)[c], 2009(ST4)[c], 2010(SM2)[c], 2011(SM2)[c], 2012(HY4)[c], 2013(PO2)[c].

MAY: 2014(AT2), 2015(AT2), 2016(AT2), 2017(HY5), 2018(HY5), 2019(HY5), 2020(HY5), 2021(HY5), 2022(HY5), 2023(PL2), 2024(PL2), 2025(PL2), 2026(PP1), 2027(PP1), 2028(PP1), 2029(PP1), 2030(SA3), 2031(SA3), 2032(SA3), 2033(SA3), 2034(ST5), 2035(ST5), 2036(ST5), 2037(ST5), 2038(PL2), 2039(PL2), 2040(AT2)[c]. 2041(PL2)[c], 2042(PP1)[c], 2043(ST5)[c], 2044(SA3)[c], 2045(HY5)[c], 2046(PP1), 2047(PP1).

JUNE: 2048(CP1), 2049(CP1), 2050(CP1), 2051(CP1), 2052(CP1), 2053(CP1), 2054(CP1), 2055(CP1), 2056 (HY6), 2057(HY6), 2058(HY6), 2059(IR2), 2060(IR2), 2061(PO3), 2062(SM3), 2063(SM3), 2064(SM3), 2065 (ST6), 2066(WW2), 2067(WW2), 2068(WW2), 2069(WW2), 2070(WW2), 2071(WW2), 2072(CP1)[c], 2073(IR2)[c], 2074(PO3)[c], 2075(ST6)[c], 2076(HY6)[c], 2077(SM3)[c], 2078(WW2)[c].

JULY: 2079(HY7), 2080(HY7), 2081(HY7), 2082(HY7), 2083(HY7), 2084(HY7), 2085(HY7), 2086(SA4), 2087 (SA4), 2088(SA4), 2089(SA4), 2090(SA4), 2091(EM3), 2092(EM3), 2093(EM3), 2094(EM3), 2095(EM3), 2096 (EM3), 2097(HY7)[c], 2098(SA4)[c], 2099(EM3)[c], 2100(AT3), 2101(AT3), 2102(AT3), 2103(AT3),2104(AT3), 2105(AT3), 2106(AT3), 2107(AT3), 2108(AT3), 2109(AT3), 2110(AT3), 2111(AT3), 2112(AT3),2113(AT3), 2114(AT3), 2115(AT3), 2116(AT3), 2117(AT3), 2118(AT3), 2119(AT3), 2120(AT3), 2121(AT3),2122(AT3), 2123(AT3), 2124(AT3), 2125(AT3).

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NOVEMBER: 2241(HY11), 2242(HY11), 2243(HY11), 2244(HY11), 2245(HY11), 2246(SA6), 2247(SA6), 2248 (SA6), 2249(SA6), 2250(SA6), 2251(SA6), 2252(SA6), 2253(SA6), 2254(SA6), 2255(SA6), 2256(ST9), 2257(ST9), 2258(ST9), 2259(ST9), 2260(HY11), 2261(ST9)[c], 2262(ST9), 2263(HY11), 2264(ST9), 2265(HY11), 2266(SA6), 2267(SA6), 2268(SA6), 2269(HY11)[c], 2270(ST9).

DECEMBER: 2271(HY12)[c], 2272(CP2), 2273(HW4), 2274(HW4), 2275(HW4), 2276(HW4), 2277(HW4), 2278 (HW4), 2279(HW4), 2280(HW4), 2281(IR4), 2282(IR4), 2283(IR4), 2284(IR4), 2285(PO6), 2286(PO6), 2287 (PO6), 2288(PO6), 2289(PO6), 2290(PO6), 2291(PO6), 2292(SM6), 2293(SM6), 2294(SM6), 2295(SM6), 2296 (SM6), 2297(WW4), 2298(WW4), 2299(WW4), 2300(WW4), 2301(WW4), 2302(WW4), 2303(WW4), 2304(HW4), 2305(ST10), 2306(CP2), 2307(CP2), 2308(ST10), 2309(CP2), 2310(HY12), 2311(HY12), 2312(PO6), 2313(PO6), 2314(ST10), 2315(HY12), 2316(HY12), 2317(HY12), 2318(WW4), 2319(SM6), 2320(SM6), 2321(ST10), 2322 (ST10), 2323(HW4)[c], 2324(CP2)[c], 2325(SM6)[c], 2326(WW4)[c], 2327(IR4)[c], 2328(PO6)[c], 2329(ST10)[c], 2330 (CP2).

c. Discussion of several papers, grouped by divisions.

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DECEMBER 1959 — 45 NO. ST 10
VOLUME 85 PART 2

Your attention is invited

NEWS OF THE STRUCTURAL DIVISION OF ASCE

JOURNAL OF THE STRUCTURAL DIVISION
PROCEEDINGS OF THE AMERICAN SOCIETY OF CIVIL ENGINEERS

DIVISION ACTIVITIES
STRUCTURAL DIVISION

Proceedings of the American Society of Civil Engineers

NEWS

December, 1959

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COMMITTEE ON MASONRY AND REINFORCED CONCRETE

The Administrative Committee of the Committee on Masonry and Reinforced Concrete under chairman, Phil M. Ferguson met at the Palmer House in Chicago on September 9, 1959. The Committee's function is mostly one of coordinating, establishing priorities, exploring needs and in general expediting task committee work.

The Committee on Folded Plate Construction has found it difficult to put together a manual on folded plates since there is a substantial difference within the committee on the matter of techniques of theory and design, some more exact than others. These are differences in points of view rather than basic differences. The Committee is in agreement on practice and construction procedures. A report, which will consist of signed chapters sponsored by individual committee members, is at present being assembled.

Progress of the Joint Committee on Limit Design has been slow because of the need for developing background material. The committee is not satisfied simply to adapt from plastic design in steel. A three-year research program, primarily on the ductility of joints, has been set up at the University of Illinois (by Portland Cement Association).

Note: No. 1959-45 is part of the copyrighted Journal of the Structural Division, Proceedings of the American Society of Civil Engineers, Vol. 85, ST 10, December, 1959.

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After a number of years of study and many sponsored research projects, the work of the Joint Committee on Shear and Diagonal Tension is headed in the direction of a final report. The committee will report its findings at the March ACI meeting and hopes for a final report in 1961. The report will cover beams with and without stirrups, slabs, provisions for torsion, and annotated bibliography.

The Joint Committee on Design of Reinforced Concrete Slabs, through sponsored research at the University of Illinois, is in the midst of a program involving testing of five quarter scale floors, each of nine slab panels. Flat plate and flat slab specimens have been tested, two-way slabs on beams are now ready for test and there remain two specimens not yet started. Those remaining are two-way slabs on shallow beams and a flat plate with wire mesh reinforcement.

The Committee on Reinforced Masonry Design and Practice reports a number of panel shear tests of reinforced masonry have recently been made in California. These involve panels of both brick and hollow block units, varying steel percentages, grouting and mortars. The committee has the design section of its report nearly ready for review and the construction practices section is nearly complete.

The ASCE-ACI Joint Committee on Composite Construction held a meeting during the ASCE Convention in Washington, D.C. The major portion of the discussion was concerned with tentative recommendations for composite designs for buildings. A draft was presented by a Task Committee headed by P. P. Page, Jr. The recommendations, explanations and an introductory statement will constitute the first progress report of the Committee. During the same convention, the Committee sponsored a technical session on the theme "Composite Design in Building Construction." Four papers were presented on design, research and construction.

Fundamental Research in Plain Concrete

The Second National Conference on Fundamental Research in Plain Concrete, which was to have been held next year has been postponed until the fall of 1961. Other important conferences such as the Fourth Symposium on the Chemistry of Cement and the research meetings of the American Society of Civil Engineers scheduled for the fall of 1960 made it desirable to delay the Concrete Conference.

The Second National Conference on Fundamental Research in Plain Concrete will be conducted in a manner similar to the highly successful First Conference, held September, 1958.

Tentative plans are to hold the conference at the spacious and attractive Allerton Park Conference Center of the University of Illinois on September 5 to 8, 1961. The discussions throughout the conference will be maintained on a fundamental level and the topics considered will be more strictly defined than the general topics discussed in the First Conference.

The sponsors of the Second Conference and their representatives, which constitute the Conference Steering Committee are:

American Concrete Institute . . . . . Bryant Mather
American Society for Testing Materials . . . . . K. B. Woods
American Society of Civil Engineers,
Structural Division . . . . . Leo H. Corning

Portland Cement Association . . . . . Hubert Woods
Reinforced Concrete Research Council . . . . . Gene M. Nordby
University of Illinois . . . . . Clyde E. Kesler

Attendance at the Conference will be by invitation. Anyone desiring information about the conference should write Professor Clyde E. Kesler, Department of Theoretical and Applied Mcehanics, University of Illinois, Urbana, Illinois.

CONFERENCE ON ELECTRONIC COMPUTATION

The Second Structural Division Conference on Electronic Computation is scheduled for 8-9 September 1960 at the new Pittsburgh Hilton Hotel. A preliminary announcement of this conference was published in the September issue of the NEWSLETTER.

The organizing Committee wishes to inform prospective speakers that complete manuscripts are due 15 April 1960. Abstracts of papers were due on 30 November 1959. Final selections will be made from the completed manuscripts before 15 June 1960.

Courses in Electronic Computation

In addition to courses enumerated in the September NEWSLETTER, The Illinois Institute of Technology is currently offering evening courses titled "Introduction to Digital Computing" (Math 457), "Automatic Computers for Business Systems" (B.E. 426), and "Operations Research I" (I.E. 455). A course, "Introduction to Differential Analyzers" (Math 458), is given as a sequel to Math 457.

This past summer the University of New Mexico offered a course, "Introduction to Digital Computers" covering computer logic, coding, computer organization, and programming; calculus and senior standing were prerequisites.

Replies to the Task Committee on Educational Aspects of Electronic Computation's questionaire indicate that the range of computer usuage for teaching within Civil Engineering departments varies from nil to requiring a course in computer techniques prior to graduation. A number of schools make use of computers to provide solutions to specific problems given in the undergraduate curriculum, thus extending the range of problems which the student encounters. Others give courses in the application of the computer to civil engineering problems, in addition to (or in place of) the usual mathematics courses in which coding is taught.

COLORADO SECTION - STRUCTURAL DIVISION

Eight meetings were held by the Colorado Section - Structural Division during the period September, 1958 through May, 1959. The subjects and speakers for the meetings were as follows:

Engineers-Contractors Relationship by C. Schrepferman
Matrix Analysis of Frames by W. L. Menoher
Model Analysis of Structures by K. H. Gerstle and A. Knott
Reinforced Concrete in Russia by W. H. Price

Cold-Worked Reinforcing Bars by I. F. Jorgensen
Design of Geln Canyon Bridge by R. Sailer
Use of Electronic Computers in Civil Engineering by F. Swain
Modern Methods of Continuous Bridge Analysis by T. V. Stradley.

The above program is repeated here to show what a local section with a Structural Division can accomplish. The total expenditure for the year was only $21 for postcards. Perhaps other local sections of the ASCE would do well to have a special night set aside each month for a Structural Division meeting.

The officers for the current year of the Colorado Section-Structural Division are:

Chairman	E. Vernon Konkel
Vice Chairman	John W. O'Hara
Secretary-Treasurer	James Chinn

COMMENTARY ON PLASTIC DESIGN IN STEEL

Readers of the Structural Division Newsletter will remember a note in the September issue about the Commentary on Plastic Design in Steel. This Commentary is the work of a Joint Committee of Welding Research Council and the American Society of Civil Engineers. The purpose of this Commentary is to provide the background of theory and tests upon which recommendations may be based for design by the plastic method. The first two progress reports appeared in the July issue of the Engineering Mechanics Division JOURNAL, ASCE.

Progress Reports 3 and 4 appear in the October issue of the JOURNAL. They are concerned with "Additional Design Considerations"—factors such as shear force, local buckling of flanges and webs, lateral stability, and repeated loading. Additional progress reports will appear in later issues.

The Joint Committee has approved publication of the Commentary with a strong recommendation that it be read carefully by all structural engineers.

SYMPOSIUM ON PLASTICITY

The Second Symposium on Naval Structural Mechanics will be held April 5, 6, and 7, 1960 at Brown University, Providence, R. I., under the joint sponsorship of the Office of Naval Research, Department of the Navy, and of Brown University. The Symposium will be devoted exclusively to the field of plasticity. The program will consist of critical surveys in selected areas and of reports on original research, with ample time for discussion. The organizing committee consists of Professors E. H. Lee and P. S. Symonds (co-chairmen), Professors D. C. Drucker and W. Prager.

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News items for the February issue of the Newsletter should be submitted to the editor before the end of December. Comments on the Newsletter and suggestions for its improvement will be welcomed.

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